



Q1. If vectors \vec{a} and \vec{b} are such that $|\vec{a}| = \frac{1}{2}$, $|\vec{b}| = \frac{4}{\sqrt{3}}$ and $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$, then find $|\vec{a} \cdot \vec{b}|$.

Answer: We know that $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \hat{n}$

$$\text{Or } |\vec{a} \times \vec{b}| = ab \sin\theta$$

$$\text{Or } \frac{1}{\sqrt{3}} = \frac{1}{2} \times \frac{4}{\sqrt{3}} \sin\theta$$

$$\text{Or } \sin\theta = \frac{1}{2}$$

$$\text{We know that } |\vec{a} \cdot \vec{b}| = |\vec{a}||\vec{b}| \cos\theta = \frac{1}{2} \times \frac{4}{\sqrt{3}} \times \sqrt{1 - \sin^2\theta} = \frac{1}{2} \times \frac{4}{\sqrt{3}} \times \sqrt{1 - \frac{1}{4}} = \frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$$

Q2. If \vec{a} and \vec{b} are unit vectors, then what is the angle between \vec{a} and \vec{b} for $\vec{a} - \sqrt{2}\vec{b}$ to be a unit vector?

Answer: Given $|\vec{a}| = |\vec{b}| = 1$

$$\text{Also } |\vec{a} - \sqrt{2}\vec{b}| = 1$$

$$\text{Or } |\vec{a} - \sqrt{2}\vec{b}|^2 = 1$$

$$\text{Or } a^2 + 2b^2 - 2\sqrt{2}\vec{a} \cdot \vec{b} = 1$$

$$\text{Or } 1 + 2 - 2\sqrt{2}ab\cos\theta = 1$$

$$\text{Or } 2\sqrt{2}\cos\theta = 2$$

$$\text{Or } \cos\theta = \frac{1}{\sqrt{2}}$$

$$\text{Or } \theta = 45^\circ$$