Q26. Show that the differential equation $2ye^{x/y}dx + \left(y - 2xe^{x/y}\right)dy = 0$ is homogeneous.

Find the particular solution of this differential equation, given that x = 0 when y = 1.

Answer: Given equation $2ye^{x/y}dx + \left(y - 2xe^{x/y}\right)dy = 0$

$\frac{dx}{dy} = \frac{2xe^{x/y}-y}{2ye^{x/y}} \quad \rightarrow (1)$

Let $f(x,y) = \frac{2xe^{x/y}-y}{2ye^{x/y}}$

Then $f(\lambda x, \lambda y) = \frac{\left[2\lambda x e^{\frac{\lambda x}{\lambda y}} - \lambda y\right]}{\left[2\lambda y e^{\frac{\lambda x}{\lambda y}}\right]} = \frac{\lambda\left[2xe^{\frac{x}{y}} - y\right]}{\lambda\left[2ye^{\frac{x}{y}}\right]} = \lambda^0[f(x,y)]$

Thus, $f(x,y)$ is a homogeneous function of degree zero.
Therefore, the given differential equation is a homogeneous differential equation.
Let $x = vy$ now differentiating with respect to y, we get
$\frac{dx}{dy} = v + y\frac{dv}{dy}$

Putting in equation (1) values of x and $\frac{dx}{dy}$, we get

$v + y\frac{dv}{dy} = \frac{2vye^v - y}{2ye^v} = \frac{2ve^v - 1}{2e^v}$

$or,\ y\frac{dv}{dy} = \frac{2ve^v-1}{2e^v} - v \ \ or,\ y\frac{dv}{dy} = -\frac{1}{2e^v}$

$or,\ 2e^v dv = -\frac{dy}{y} \ \ or,\ \int 2e^v\, dv = -\int \frac{dy}{y}$

$or,\ 2e^v = -\log|y| + K$

Substituting the value of v, we get

$2e^{\frac{x}{y}} + \log|y| = K \rightarrow (2)$

Substituting $x = 0$ and $y = 1$ in equation (2), we get

$2e^0 + \log|1| = K \ or\ K = 2$

Substituting the value of K in equation (2), we get

$2e^{\frac{x}{y}} + \log|y| = 2$, Which is the particular solution of the given differential equation.

Q27. Find the vector equation of the plane passing through three points with position vectors $\hat{\imath}+\hat{\jmath}-2\hat{k},\ 2\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\hat{\imath}+2\hat{\jmath}+\hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{\imath}-\hat{\jmath}-\hat{k}+\lambda(2\hat{\imath}-2\hat{\jmath}+\hat{k})$.

Answer:

Let

$\vec{a}=\hat{\imath}+\hat{\jmath}-2\hat{k}$

$\vec{b}=\ 2\hat{\imath}-\hat{\jmath}+\hat{k}$ and

$\vec{c}=\ \hat{\imath}+2\hat{\jmath}+\hat{k}$.

be the position vectors of three points.

Therefore the equation of the plane passing through the points $\vec{a},\vec{b}$ and $\vec{c}$

$(\vec{r}-\vec{a}).\left[(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\right]=0$

$or\ \left[\vec{r}-(\hat{\imath}+\hat{\jmath}-2\hat{k})\right].\left[(\hat{\imath}-3\hat{\jmath})\times(\hat{\jmath}+3\hat{k})\right]=0$

$or\ \left[\vec{r}-(\hat{\imath}+\hat{\jmath}-2\hat{k})\right].(\hat{k}-3\hat{\jmath}-9\hat{\imath})=0$

$or\ \vec{r}.(-9\hat{\imath}-3\hat{\jmath}+\hat{k})+14=0$

$or\ \vec{r}.(9\hat{\imath}+3\hat{\jmath}-\hat{k})=14\quad\rightarrow(1)$

Now vector equation of the plane

$\vec{r}.(9\hat{\imath}+3\hat{\jmath}-\hat{k})=14$

The equation of the given line is

$\vec{r}=(3\hat{\imath}-\hat{\jmath}-\hat{k})+\lambda(2\hat{\imath}-2\hat{\jmath}+\hat{k})$

Position vector of any point on the given line is

$\vec{r}=(3+2\lambda)\hat{\imath}+(-1-2\lambda)\hat{\jmath}+(-1+\lambda)\hat{k}\quad\rightarrow(2)$

Since this point lines in plane stated in equation (1)

$\left[(3+2\lambda)\hat{\imath}+(-1-2\lambda)\hat{\jmath}+(-1+\lambda)\hat{k}\right].(9\hat{\imath}+3\hat{\jmath}-\hat{k})=14$

$or\ 9(3+2\lambda)+3(-1-2\lambda)-(-1+\lambda)=14$

$or\ 11\lambda+25=14$

$or\ \lambda=-1$

Putting value λ in equation(1)

$\vec{r}=(3+2\lambda)\hat{\imath}+(-1-2\lambda)\hat{\jmath}+(-1+\lambda)\hat{k}$

$=(3+2(-1))\hat{\imath}+(-1-2(-1))\hat{\jmath}+(-1+(-1))\hat{k}$

$=\hat{\imath}+\hat{\jmath}-2\hat{k}$

Thus coordinates of the point of intersection of this plane and the line is are (1, 1, -2)

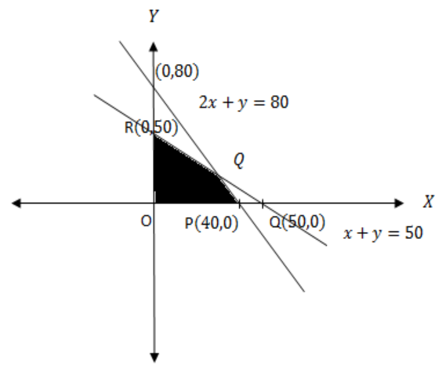
vector equation of the plane

$\vec{r}=\hat{\imath}+\hat{\jmath}-2\hat{k}$

Q28. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment?

Answer:

Let the x= Land allocated for crop A and
y= Land allocated for crop B.
Maximum area of the land available for two crops is 50 hectares.
$\therefore x + y \leq 50$
Liquid herbicide to be used for crops A and B are at the rate of 20 litres and 10 litres per hectare respectively. Maximum amount of herbicide to be used is 800 litres.
$so, 20x + 10y \leq 800$
$or\ 2x + y \leq 80$
The profits from crops A and B per hectare are Rs 10500 and Rs 9000 respectively.
Hence total profit =
Rs $(10500x + 9000y)$ = Rs 1500 $(7x + 6y)$
Therefore LPP for the above statement can be given as:
Maximize $Z = 1500 \times (7x + 6y)$
subject to the constraints
$x + y \leq 50 \quad \rightarrow (1)$
$2x + y \leq 80 \quad \rightarrow (2)$
$x \geq 0, y \geq 0$



For plotting graph:
$\frac{x}{50} + \frac{y}{50} = 1$ Intercept on X and Y axis (50, 0) and (0, 50) obtained also $\frac{x}{40} + \frac{y}{80} = 1$, X-Y intercept (40,0) and (0,80) obtained.
Solving (1) and (2):$x = 30, y = 20\ So\ Q(30,20)$
Feasible region shaded having corner points O(0,0),P(40,0),Q(30,40),R(0,50)
$Z_{O(0,0)} = 0$
$Z_{P(40,0)} = 1500 \times 280 = 420000$
$Z_{Q(30,20)} = 1500 \times (210 + 120) = 495000$
$Z_{R(0,50)} = 1500 \times 300 = 450000$
This Z is maximum at Q: $Z_{Q(30,20)} = 495000$
Maximum Profit:495000
x= Land for crop A=30 hectares
y=Land for crop B= 20 hectares

Yes I agree protection of wild life is necessary for balance in environment.

Q29. Assume that the chance of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation.

Answer: Let A = Event that person has heart attack

E_1 = Event the person follows Yoga course and meditation

E_2 = Event that the person follows drug prescription

Given: $\mathrm{P(A)} = 0.40$

$\mathrm{P(E_1)} = \mathrm{P(E_2)} = \frac{1}{2}$ [At a time a patient can choose any one of the two options with equal probabilities.]

$\mathrm{P(A/E_1)} = 0.40 \times 0.70 = 0.28$ [yoga and meditation reduces chance by 30%, so 70% chance exists]

$\mathrm{P(A/E_2)} = 0.40 \times 0.75 = 0.30$ [drug and prescription reduces chance by 25%,so 75% chance exists.]

Probability that the patient suffering a heart attack followed a course of meditation and yoga

$$\mathrm{P(E_1/A)} = \frac{\mathrm{P(E_1)\,P(A/E_1)}}{\mathrm{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)}}$$

$$= \frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28+\frac{1}{2}\times 0.30}$$

$$= \frac{28}{28+30} = \frac{28}{58} = \frac{14}{29}$$

$$\text{Now, } \mathrm{P(E_2/A)} = \frac{\mathrm{P(E_2)\,P(A/E_2)}}{\mathrm{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)}}$$

$$= \frac{\frac{1}{2}\times 0.30}{\frac{1}{2}\times 0.28+\frac{1}{2}\times 0.30}$$

$$= \frac{30}{28+30} = \frac{30}{58} = \frac{15}{29}$$

Hence we find $\mathrm{P(E_1/A)} < P(\mathrm{E_2/A})$, therefore course of yoga and meditation is more beneficial for a patient than drug and prescription.