



Q8. If a unit vector \vec{a} makes angle $\frac{\pi}{3}$ with \hat{i} , $\frac{\pi}{4}$ with \hat{j} and acute angle θ with \hat{k} , then find the value of θ .

Answer: Considering $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ be the unit vector.

$$\therefore x = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$y = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$z = \cos \theta$$

Since a is unit vector therefore

$$\text{Now, } x^2 + y^2 + z^2 = 1$$

$$\text{or } \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 \theta = 1$$

$$\text{or } \cos^2 \theta = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$$

$$\therefore \cos \theta = \frac{1}{2} \text{ or } \theta = \frac{\pi}{3}$$

Q9. Find the Cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

Answer: The equation of the given line is:

$$\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$$

$$\text{i. e., } \frac{x+3}{3} = \frac{y-4}{-5} = \frac{z+8}{6}$$

We know that the direction ratio of parallel lines is same.

Therefore direction ratios of the required line are 3, -5, and 6.

Now the equation of the straight line passing through $(-2, 4, -5)$ and having direction ratios 3, -5, 6 is

$$\frac{x-2}{3} = \frac{y-4}{-5} = \frac{z-5}{6}$$

$$\text{or } \frac{x+2}{3} = \frac{4-y}{5} = \frac{z+5}{6}$$



Q10. The amount of pollution content added in air in a city due to x -diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above questions.

Answer: $P(x) = 0.005x^3 + 0.02x^2 + 30x$

Differentiating with respect to x ,

Marginal increase in pollution content = $\frac{dP(x)}{dx} = 0.015x^2 + 0.04x + 30$ (i)

Putting $x=3$ in (i), $\left(\frac{dP(x)}{dx}\right)_{x=3} = 0.015 \times 9 + 0.04 \times 3 + 30 = 30.255$

Therefore, the value of marginal increase pollution content is 30.255.

Increment of 3 diesel vehicle increases the pollution by 30.255. For environment safe earth we need to decrease pollution content in air by reducing the number of diesel vehicle.