5.Find the differential equation representing the family of curves $V = \frac{A}{r} + B$, where A and B are arbitrary constants.

Answer: Given $V = \frac{A}{r} + B$

Differentiating with respect to r we get

$$\frac{dv}{dr} = -\frac{A}{r^2} + 0$$

$$or\ A = -r^2 \times \frac{dv}{dr}$$

Differentiating again with respect to r we get

$$0 = -\left(2r\frac{dv}{dr} + r^2\frac{d^2v}{dr^2}\right)$$

$$or\ r^2\frac{d^2v}{dr^2} + 2r\frac{dv}{dr} = 0$$

6. Find the integrating factor of the differential equation

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$$

Answer:

We know for differential equation of the form $\frac{dy}{dx} + Py = Q \rightarrow (1)$ Integrating factor (IF)= $I = e^{\int Pdx}$ Given equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$	$or\ \frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$ $or\ \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$ Comparing with equation(1) : $P = \frac{1}{\sqrt{x}}$ Therefore Integrating factor (IF)= $I = e^{\int Pdx}$ $I = e^{\int Pdx} = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$

7. If $A=\begin{pmatrix}2&0&1\\2&1&3\\1&-1&0\end{pmatrix}$ find $A^2-5A+4I$ and hence find a matrix X such that $A^2-5A+4I+X=0$

Answer : A^2=$\begin{pmatrix}2&0&1\\2&1&3\\1&-1&0\end{pmatrix}\begin{pmatrix}2&0&1\\2&1&3\\1&-1&0\end{pmatrix}$

$$=\begin{pmatrix}2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times(-1) & 2\times1+0\times3+1\times0\\2\times2+1\times2+3\times1 & 2\times0+1\times1+3\times(-1) & 2\times1+1\times3+3\times0\\1\times2+(-1)\times2+0\times1 & 1\times0+(-1)\times1+0\times(-1) & 1\times1+(-1)\times3+0\times0\end{pmatrix}$$

$$=\begin{pmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{pmatrix}$$

$$A^2-5A+4I=\begin{pmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{pmatrix}-5\begin{pmatrix}2&0&1\\2&1&3\\1&-1&0\end{pmatrix}+4\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$=\begin{pmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{pmatrix}-\begin{pmatrix}10&0&5\\10&5&15\\5&-5&0\end{pmatrix}+\begin{pmatrix}4&0&0\\0&4&0\\0&0&4\end{pmatrix}=\begin{pmatrix}-1&-1&-3\\-1&-3&-10\\-5&4&2\end{pmatrix}=-1\begin{pmatrix}1&1&3\\1&3&10\\5&-4&-2\end{pmatrix}$$

Therefore

$$A^2-5A+4I=-1\begin{pmatrix}1&1&3\\1&3&10\\5&-4&-2\end{pmatrix}$$

Also we find $A^2-5A+4I+\begin{pmatrix}1&1&3\\1&3&10\\5&-4&-2\end{pmatrix}=0$

Comparing with $A^2-5A+4I+X=0$

Answer

$$\boldsymbol{A^2-5A+4I=-1\begin{pmatrix}1&1&3\\1&3&10\\5&-4&-2\end{pmatrix}}$$

$$\boldsymbol{X=\begin{pmatrix}1&1&3\\1&3&10\\5&-4&-2\end{pmatrix}}$$

OR if $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}, find\ (A')^{-1}$

Answer: $|A| = \begin{vmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{vmatrix} = 1(-1-8) - 2(0+8) + 3(0-2) = -9 + 16 - 6 = 1$

$hence\ |A| \neq 0$

We know that $A^{-1} = \frac{Adj.A}{|A|}$

To find Adj. A, let us find Cofactors of elements

$C_{11} = \begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} = -1 - 8 = -9$	$C_{12} = -\begin{vmatrix} 0 & 4 \\ -2 & 1 \end{vmatrix} = -8$	$C_{13} = \begin{vmatrix} 0 & -1 \\ -2 & 2 \end{vmatrix} = -2$
$C_{21=} - \begin{vmatrix} -2 & 3 \\ 2 & 1 \end{vmatrix} = 2 + 6 = 8$	$C_{22} = \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = 7$	$C_{23} = -\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = 2$
$C_{31=} \begin{vmatrix} -2 & 3 \\ -1 & 4 \end{vmatrix} = -8 + 3 = -5$	$C_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -4$	$C_{33} = \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} = -1$

We know that $A^{-1} = \frac{Adj.A}{|A|} = \frac{1}{1}\begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}$

Required $(A')^{-1} = (A^{-1})' = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

8. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of $f(2x) - f(x)$

Answer: Multiplying R_2 by x

$$f(x) = \frac{1}{x}\begin{vmatrix} a & -1 & 0 \\ ax^2 & ax & -x \\ ax^2 & ax & a \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$

$$f(x) = \frac{1}{x}\begin{vmatrix} a & -1 & 0 \\ ax^2 & ax & -x \\ 0 & 0 & a+x \end{vmatrix}$$

$or\ f(x) = \frac{1}{x}(a+x)(a^2x + ax^2)$

$or\ f(x) = a(a+x)(a+x)$

$or\ f(x) = a(a+x)^2 \to (1)$

Also $f(2x) = a(a+2x)^2 \to (2)$

$f(2x) - f(x) = \ a(a+2x)^2 - a(a+x)^2$

$= a(a^2 + 4x^2 + 4ax - a^2 - x^2 - 2ax)$

$= a(3x^2 + 2ax)$