9. Find : $\int \frac{dx}{sinx+sin2x}$

Answer: $I = \int \frac{dx}{sinx+sin2x} = \int \frac{dx}{sinx+2sinxcosx} = \int \frac{dx}{sinx(1+2cosx)} = \int \frac{sinxdx}{sin^2x(1+2cosx)} = \int \frac{sinxdx}{(1-cos^2x)(1+2cosx)} =$

$$or\ I = \int \frac{sinxdx}{(1-cosx)(1+cosx)(1+2cosx)}$$

$Putting\ cosx = u\ , -sinxdx = du$

$$I = \int \frac{-du}{(1-u)(1+u)(1+2u)}$$

Let $\frac{1}{(1-u)(1+u)(1+2u)} = \frac{A}{1-u} + \frac{B}{1+u} + \frac{C}{1+2u}$

$or\ 1 = A(1+u)(1+2u) + B(1-u)(1+2u) + C(1-u)(1+u)$

Putting $u = 1, A = \frac{1}{6}$

Putting $u = -1, B = -\frac{1}{2}$

Putting $u = -\frac{1}{2}, C = \frac{4}{3}$

$$\frac{1}{(1-u)(1+u)(1+2u)} = \frac{1}{6(1-u)} - \frac{1}{2(1+u)} + \frac{4}{3(1+2u)}$$

$$I = \int -\left[\frac{1}{6(1-u)} - \frac{1}{2(1+u)} + \frac{4}{3(1+2u)}\right] du$$

$$or\ I = \frac{1}{6} log|1-u| + \frac{1}{2} log|1+u| - \frac{2}{3} \times log|1+2u| + C$$

$$or\ \boldsymbol{I = \frac{1}{6} log|1-cosx| + \frac{1}{2} log|1+cosx| - \frac{2}{3} \times log|1+2cosx| + C}$$

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OR Integrate the following with respect to x $\frac{x^2-3x+1}{\sqrt{1-x^2}}$

Answer: $I = \int \frac{x^2-3x+1}{\sqrt{1-x^2}}dx$

$$\frac{x^2-3x+1}{\sqrt{1-x^2}} = -\frac{-x^2+3x-1}{\sqrt{1-x^2}} = -\frac{1-x^2+3x-2}{\sqrt{1-x^2}} = -\left(\frac{1-x^2}{\sqrt{1-x^2}} + \frac{3x}{\sqrt{1-x^2}} - \frac{2}{\sqrt{1-x^2}}\right)$$

$$I = -\left(\int \sqrt{1-x^2}\,dx - \frac{3}{2}\int \frac{-2x}{\sqrt{1-x^2}}dx - 2\int \frac{1}{\sqrt{1-x^2}}dx\right)$$

$$I = -\left(I_1 - \frac{3\sqrt{1-x^2}}{2\times\frac{1}{2}} - 2sin^{-1}x\right) + C_1 \rightarrow (1)$$

$$where\ I_1 = \int \sqrt{1-x^2}\,dx$$

Putting $x = sin\theta, dx = cos\theta d\theta$

$$or\ \ I_1 = \int cos\theta \times\ cos\theta d\theta = \int \frac{1+cos2\theta}{2}d\theta = \frac{1}{2}\theta + \frac{1}{4}sin2\theta = \frac{1}{2}sin^{-1}x + \frac{1}{4}\times 2x\sqrt{1-x^2} + C_2 \rightarrow (2)$$

Putting (2) in (1) we get

$$I = -\left(\frac{1}{2}sin^{-1}x + \frac{1}{4}\times 2x\sqrt{1-x^2} + C_2 - \frac{3\sqrt{1-x^2}}{2\times\frac{1}{2}} - 2sin^{-1}x\right) + C_1$$

$$I = -\frac{1}{2}sin^{-1}x - \frac{1}{4}\times 2x\sqrt{1-x^2} + \frac{3\sqrt{1-x^2}}{2\times\frac{1}{2}} + 2sin^{-1}x + C$$

$$\boldsymbol{I = \frac{3}{2}sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} + C}$$