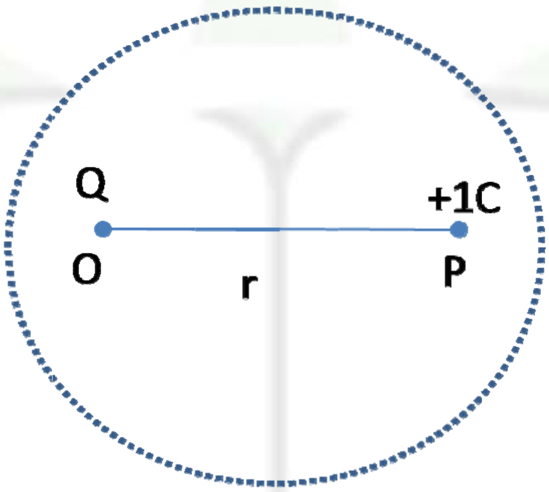
Electric field: If we keep a charged particle at any point in space the property of the medium surrounding the charged particle changes in a sense that if another charged particle is kept in that surrounding it experiences a force known as electric field of the first charged particle.

The strength of the electric field at a point in space can be represented in two different ways

(1) Electric Intensity

(2) Electric Potential

Electric Intensity : The electric intensity at a point is defined as the force experienced by a real or imaginary unit positive charge (test charge) kept at that point.



A point charge Q is kept at O. P is any point in the surrounding OP=r , let us imagine 1C positive charge at P.

The force experienced by that imaginary unit (+)ve charge

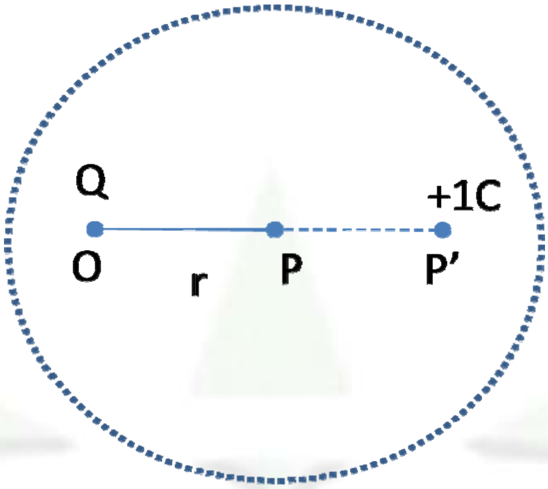
$$= \frac{Q}{4\pi\varepsilon_0 r^2} \text{ along } \overrightarrow{OP} = \text{Intensity at P}$$

$$E == \frac{Q}{4\pi\varepsilon_0 r^2} \text{ volt}/\text{meter}$$

If E is the electrical intensity at a point in the field then if we keep a charge Q' at that point the force experienced by that point = EQ'

Intensity is a vector quantity. If the given charge is (+) ve intensity is away from the given charge and if the given charge is (-)ve then intensity is towards the given charge.

Electrical Potential: Electric potential at a point in the electric field is defined as the work done to bring a unit (+) ve charge (test charge) from infinity up to that point.



A point charge Q is kept at O, if a charge Q' is taken from P' to P since a displacement occurs in presence of force work is done. This work is stored in the charged particle Q' in the form of electrical potential energy at the point P .

"The work done to bring any charge from infinity up to any point in the electric field is stored as the electric potential energy in that charged particle at that point."

If instead of bringing any charge if we bring a unit positive charge (test charge) then the work done to bring the unit (+) ve charge from infinity up to a point in the electric field is known as electric potential at that point.

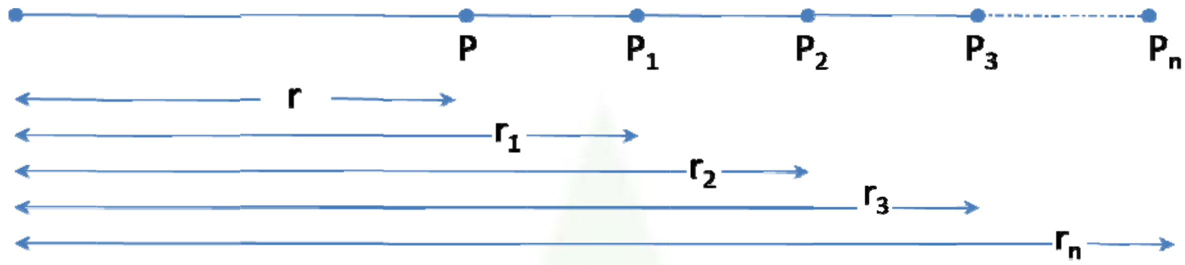
Thus electric potential energy per unit charge is the potential.

Potential = Potential energy / charge

Electrical potential energy = V x Q' (Joules = volt x coulomb)

Electric potential at a point due to a point charge:

Electric Potential: The work done to bring a unit positive charge (test charge) from infinity up to the given point in the electric field is known as electric potential at that point.



A point charge +Q is kept at O. P is the given point

$OP = r$

ε_0 = Permittivity of the surrounding space

Joining OP and producing upto any point P_n

Dividing PP_n by points $P_1, P_2, P_3, \dots P_{n-1}$ at distances $r_1, r_2, r_3 \dots r_{n-1}$

Force on +1c charge at P $= \dfrac{Q.1}{4\pi\varepsilon_0 r^2}$ along $\overrightarrow{OP}$

Force on +1c charge at $P_1 = \dfrac{Q.1}{4\pi\varepsilon_0 r_1^2}$ along $\overrightarrow{OP_1}$

Thus the average force on +1c charge kept between P and P_n can be found by taking the geometric mean as the variation is due to inverse square distance.

$$F_{PP_1} = \sqrt{\frac{Q}{4\pi\varepsilon_0 r^2} \times \frac{Q}{4\pi\varepsilon_0 r_1^2}} = \frac{Q}{4\pi\varepsilon_0 r r_1} \text{ along } PP_1$$

Hence the work done to bring a unit (+)ve charge from P_1 to P $= F_{PP_1} \times$ distance PP_1

$$W_{+1c, P \to P_1} = \frac{Q}{4\pi\varepsilon_0 r r_1}(r_1 - r) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_1}{r r_1} - \frac{r}{r r_1}\right]$$

$$W_{+1c, P \to P_1} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$\therefore$ The work done to bring +1c (+)ve charge from P_2 to P_1

$$W_{+1c, P_1 \to P} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r}\right)$$

$$W_{+1c, P_2 \to P_1} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$W_{+1c, P_n \to P_{n-1}} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_n} - \frac{1}{r_{n-1}}\right)$$

Hence the total work done to bring unit (+)ve charge from P_n to P can be obtained by adding all the work done.

$$W_{P_n \to P} = W_{+1c, P_1 \to P} + W_{+1c, P_2 \to P_1} + ... + W_{+1c, P_n \to P_{n-1}} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_n}\right)$$

Since the point P is arbitrarily choosen it can be choosen at infinity $r_n = \infty$

$$W_{\infty \to P} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

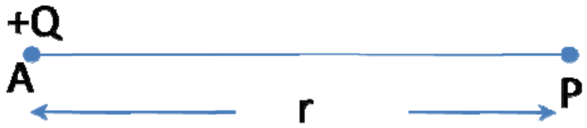
$$W_{\infty \to P} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r} = V = \text{Potential at P}$$

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Remarks: If the given charge is positive work done in bringing test charge is against the repulsive force and hence it is taken as (+) ve i.e. The potential at P due to positive charge is (+) ve.

If the given charge is negative work done in bringing the test charge in favour of the attractive force and hence it is taken as negative i.e. Potential at a point due to a negative charge is negative.

Relation between intensity and potential:



$$\text{Electric intensity at P}: E = \frac{Q}{4\pi\varepsilon_0 r^2} \rightarrow (1)$$

$$\text{Electric potential at P}: V = \frac{Q}{4\pi\varepsilon_0 r} \rightarrow (2)$$

Differentiating equation(2) w.r to r

$$\frac{dv}{dr} = \frac{Q}{4\pi\varepsilon_0}\frac{d(r^{-1})}{dr} = \frac{Q}{4\pi\varepsilon_0}\left(-r^{-1-1}\right) = -\frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

$$\frac{dv}{dr} = -E$$

$$E = -\frac{dv}{dr}$$

Thus intensity can also be defined as the rate of change of potential with distance i.e. a potential gradient with negative sign.

The negative sign indicates the potential decreases in the direction of intensity.