Moment Of Inertia

Inertia is a property inherent in every substance by which it oppose any change in its state:

(i) State of rest

(ii) State of uniform motion

Mass of the body is a measure of its inertia, smaller is the mass smaller will be the Inertia.

Let us consider rotational motion of a particle. Let a particle at rest be rotated about an axis, the state of the particle changes and the particle will oppose this change.

It is found that the opposition depends on two factors:

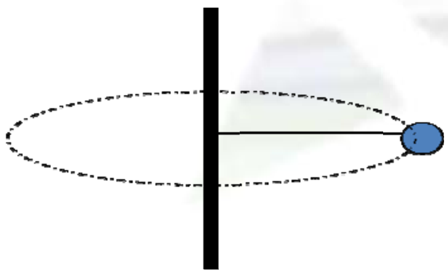
(i) Mass of the particle – greater is mass greater is opposition

(ii) The distance of the particle from the axis – greater is the distance greater will be the opposition.

Thus the rotational inertia is measured by (m.r) and the moment of this inertia about the axis of rotation is mr^2 and is known as **Moment of Inertia**.

Moment of inertia of a point mass about an axis:

$I = mr^2$ [Unit = $Kg.m^2$]



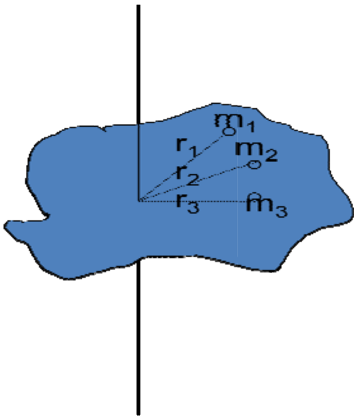
Physical significance: M.I in case of rotational motion plays exactly the same role as played by mass in case of linear motion.

* If the size of the particle is negligible compared to its distance from the axis, it is known as point mass

Table of analogy

Sr. No	Linear Motion	Rotational Motion
1	**Linear displacement [s]**	**Angular displacement [ϴ]**
2	**Linear Velocity [v = ds/dt]**	**Angular velocity [ω = dϴ/dt]**
3	**Linear Acceleration [$dv/dt = d^2s/dt^2$]**	**Angular Acceleration [$d\omega/dt = d^2\Theta/dt^2$]**
4	**Mass [m]**	**Moment of Inertia [I]**
5	**Force [$F = m. d^2s/dt^2$]**	**Torque/Moment of force [$\tau = I. d^2\Theta/dt^2$]**
6	**Linear Momentum [m.v]**	**Angular Momentum [I. ω]**
7	**Linear Kinetic Energy [½ mv^2]**	**Angular Kinetic Energy [½ I. ω^2]**

Moment of Inertia of lamina about an axis:



M = mass of the lamina

We can assume the lamina to be made up of large no of point masses m_1, m_2 , m_3.... at distances r_1,r_2,r_3 ... from the axis of rotation.

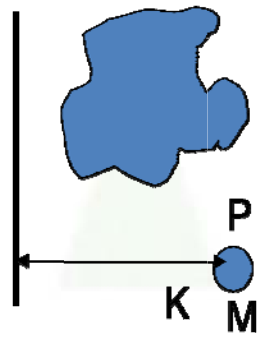
M.I of these point masses about the axis is $m_1r_1^2$, $m_2r_2^2$, $m_3r_3^2$.... respectively.

Since M.I is a scalar quantity hence their algebraic sum would give the M.I of the whole lamina about the given axis.

$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + ...$

Radius of Gyration:

Definition: The radius of gyration of a given lamina about a given axes is defined as the distance from the axis up to that point where if the whole mass of the lamina were assumed to be concentrated would gave the same M.I as with the actual distribution of the mass.



Explanation: Given m = mass of the lamina

I = M.I of the lamina about the given axis

Let us consider a point mass 'M' equal to the mass of the lamina

M.I of this point mass about the given axis

$I_1 = Mr^2$ [r = distance from the axis]

Let when the point mass M be kept at P $r = K$ & $I_1 = I$

i.e. when the point mass M is kept at P the M.I of the point mass is same as the Lamina.

i.e. $I_1 = M.K^2 = I$

Then distance 'K' is known as Radius of gyration.

Mathematically M.I can be defined as follows:

We imagine the lamina to be divided into large no of equal point masses (m) at distance r_1, r_2, r_3, ….. r_n from the axis.

$$I = mr_1^2 + mr_2^2 + mr_3^2 + \ldots\ldots\ldots\ldots + mr_n{}^2$$

$$I = m[r_1^2 + r_2^2 + r_3^2 + \ldots . + r_n^2]$$

$$I = \frac{mn[r_1^2 + r_2^2 + r_3^2 + \ldots . + r_n^2]}{n}$$

$$I = \frac{M[r_1^2 + r_2^2 + r_3^2 + \ldots . + r_n^2]}{n}$$

$$MK^2 = \frac{M[r_1^2 + r_2^2 + r_3^2 + \ldots . + r_n^2]}{n}$$

$$K = \sqrt{\frac{[r_1^2 + r_2^2 + r_3^2 + \ldots . + r_n^2]}{n}}$$

Hence radius of gyration is root mean square distance.

Radius of gyration depends on two factors:

1. Position and orientation of the axis

2. Distribution of mass about the axis

I = MK2 hence M.I of a lamina also depends on above two factors

Example: A boy stands with his arms stretched on a rotating table when he lowers down his arms the mass of the arms come close to the axis and hence radius of gyration and M.I decreases