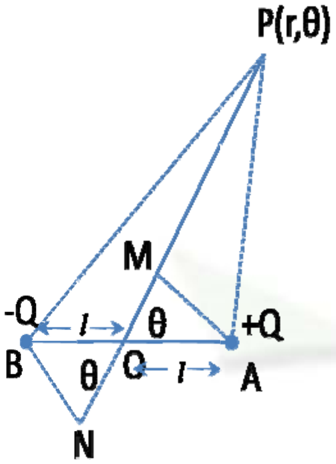
Potential and Intensity at a point due to an electric dipole:

A combination of two equal and unlike point charges kept separated by a very small distance is known as an electric dipole.



Dipolemoment= $Q \times 2l$

Where Q = charge

$2l$ = distance of separation between the charge.

$+Q$ & $-Q$ = Point charges kept at point A and B respectively

$2l = AB$ = Distance of separation between the point charges

ε_0 = Permittivity of free space surrounding the dipole.

$P(r,\theta)$ is the given point. $OP = r, \angle AOP = \theta$

Potential at P due to charge + Q at A $= \dfrac{Q}{4\pi\varepsilon_0 AP}$

Potential at P due to charge − Q at B $= -\dfrac{Q}{4\pi\varepsilon_0 BP}$

Since potential is scalar quantity total potential at P due to the dipole can be found from the algebric sum of the two

$$V = V_1 + V_2 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{AP} - \frac{1}{BP}\right] \rightarrow (1)$$

We find AP and BP interms of l, θ, r

From A & B, AM and BN are perpendiculars drawn on PO and PO produced

Since the point A and B are close, ∠APB is very small

∠APM & ∠BPM are very small

In ΔBPN; $\cos A\hat{P}M = 1 = \dfrac{PM}{AP} \therefore AP = PM = OP - OM$

From ΔBPN; $\cos B\hat{P}N = 1 = \dfrac{PN}{BP} \therefore BP = PN = OP + ON$

In ΔAOM ; $\cos\theta = \dfrac{OM}{OA}$ or $OM = l\cos\theta$

In ΔBON; $\cos\theta = \dfrac{ON}{OB}$ or $ON = l\cos\theta$

Putting the values : $AP = r - l\cos\theta, BP = r + l\cos\theta$ From equation (1)

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r + l\cos\theta - r + l\cos\theta}{(r - l\cos\theta)(r + l\cos\theta)}\right]$$

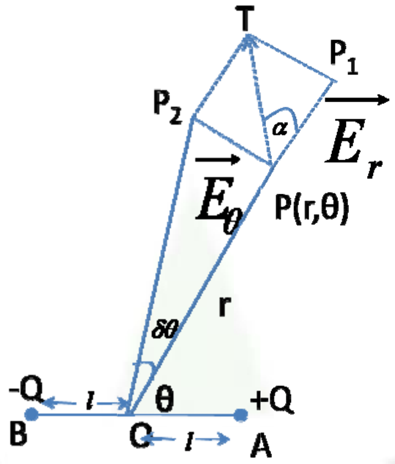
$$= \frac{Q}{4\pi\varepsilon_0}\,\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}$$

$Q.2l = P = Electric\ Dipole\ Moment$

Since l is very small, l^2 will be so small that $l^2\cos^2\theta$ can be neglected compared to r^2

$$\therefore V = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2} \rightarrow (2)$$

Equation (2) gives the potential at a point due to an electric dipole.



Keeping θ cosntant point P is taken to P_1 along $\overrightarrow{OP}$ by a very small distance dr so that potential changes by - dV

$$\therefore -\frac{dv}{dr} = \text{Potential gradient at P along OP i.e. intensity at P along OP} = \overrightarrow{E_r}$$

$$\therefore \overrightarrow{E_r} = -\frac{dv}{dr} = -\frac{d}{dr}\left[\frac{P\cos\theta}{4\pi\varepsilon_0 r^2}\right] = -\frac{P\cos\theta}{4\pi\varepsilon_0}\frac{dr^{-2}}{dr} = \frac{P\cos\theta}{4\pi\varepsilon_0 r^3} \rightarrow (3)$$

Keeping r constant point P is taken to P_2 along perpendicular to OP by a very small angle $d\theta$ so that potential changes by - dV

$$\therefore -\frac{dV}{PP_2} = \text{Potential gradient at P along perpendicular to OP} = \overrightarrow{E_\theta}$$

$$\overrightarrow{E_\theta} = -\frac{dV}{rd\theta} = -\frac{1}{r}\frac{d}{d\theta}\left[\frac{P\cos\theta}{4\pi\varepsilon_0 r^2}\right] = -\frac{P(-\sin\theta)}{4\pi\varepsilon_0 r^3} = \frac{P\sin\theta}{4\pi\varepsilon_0 r^3} \text{ along } PP_2 \rightarrow (4)$$

Hence resultant intensity at P can be found by law of parallelogram of vectors the diagonal PT represents the resultant intensity both in magnitude and direction.

$$E = \sqrt{E_r^2 + E_\theta^2} = \sqrt{\left(\frac{P}{4\pi\varepsilon_0 r^3}\right)^2\left(4\cos^2\theta + \sin^2\theta\right)} = \left(\frac{P}{4\pi\varepsilon_0 r^3}\right)\sqrt{1+3\cos^2\theta} \rightarrow (5)$$

Equation(5) gives the magnitude of intensity at P due to the dipole

$$\text{From } \Delta P_1PT : \tan\alpha = \frac{TP_1}{PP_1} = \frac{E_\theta}{E_r} = \frac{\left(\frac{P\sin\theta}{4\pi\varepsilon_0 r^3}\right)}{\left(\frac{2P\cos\theta}{4\pi\varepsilon_0 r^3}\right)} = \frac{1}{2}\tan\theta$$

$$\therefore \alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right) \rightarrow (6)$$

The direction of intensity at P due to the dipole makes an angle α with OP given by equaon(6)