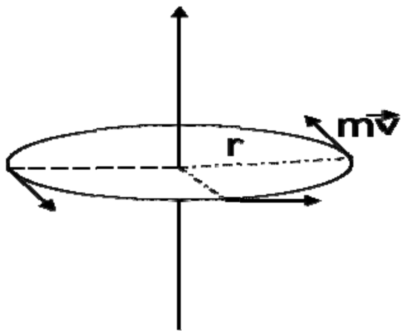
Angular Momentum

When a particle rotates about an axis the particle has a linear momentum at every point, which is along the direction of tangent to that point. The moment of this linear momentum about the axis is known as Angular momentum of the particle about the axis.



Let m = mass of the particle

v = speed of the particle along the circle

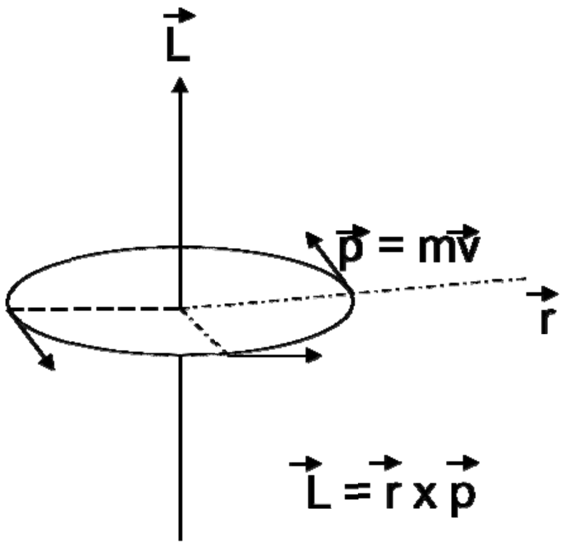
r = radius of the circle

Linear Momentum at any point p = mv

The direction of the linear momentum is along the direction of velocity i.e. along the tangent at every point. The moment of linear momentum about the axis

L = p x r = mvr

The angular momentum is a vector quantity; direction of angular momentum is perpendicular to the plane of rotation such that if we curl the fingers of right hand in the direction of rotation the thumb gives the direction of angular momentum.

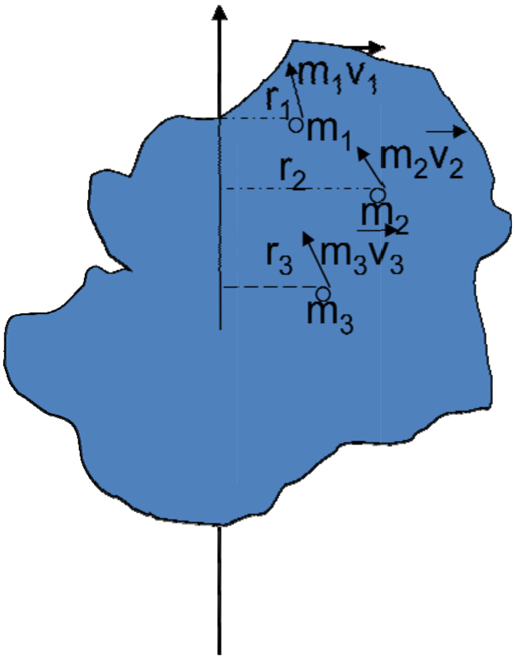


<u>We now find the angular momentum of a lamina rotating about an axis</u>

Given ω = Angular velocity of rotation of the lamina about the given axis (Constant)

I = M.I of the lamina about the given axis

We can imagine the lamina to be made up of a large number of point masses m_1, m_2, m_3 ….. at distances r_1, r_2, r_3,…. from the axis of rotation respectively.



The angular velocities of all these point masses are same equal to ω but linear velocities are different depending on their distance from the axis of rotation.

Let v_1, v_2, v_3 …….. be the linear velocities of the respective point masses.

$v_1 = r_1\,\omega$, $v_2 = r_2\,\omega$, $v_3 = r_3\,\omega$ …..

The linear momentums of these point masses are

$p_1 = m_1v_1 = m_1r_1\,\omega$, $p_2 = m_2v_2 = m_2r_2\,\omega$ ….

Taking moment of linear momenta about the axis of rotation we get the angular momentum of these point masses about the axis.

$L_1 = p_1r_1$, $L_2 = p_2r_2$, $L_3 = p_3r_3$, …….

Since all these angular momenta are in the same direction hence their sum would give the total angular momentum of the lamina about the axis of rotation.

Therefore angular momentum of the lamina about the axis of rotation

$L = L_1 + L_2 + L_3 + \ldots$

$L = p_1r_1 + p_2r_2 + p_3r_3 + \ldots$

$L = (m_1r_1\,\omega)\,r_1 + (m_1r_1\,\omega)\,r_1 + (m_1r_1\,\omega)\,r_1 + \ldots$

$L = \omega\,[m_1r_1^2 + m_1r_1^2 + m_1r_1^2 + \ldots]$

$L = \omega\,\Sigma mr^2 = \omega I$

$L = I\,\omega$