Velocity of a particle executing S.H.M

Let us consider a particle executing S.H.M along Y axis.

Given a & T = Amplitude and time period of the S.H.M

Let y be the displacement at any instant of time t, from displacement equation

$$y = aSin\omega t \longrightarrow (1)$$

Let v = The velocity of particle executing S.H.M at an instant of time t.

We know that instantaneous velocity is defined as

$$v = \frac{dy}{dt}$$
$$v = \frac{d(aSin\omega t)}{dt}$$
$$v = a\omega Cos\omega t \longrightarrow (2)$$
$$Cos\omega t = \sqrt{1 - Sin^2\omega t} = \sqrt{1 - \frac{y^2}{a^2}} = \frac{\sqrt{a^2 - y^2}}{a} \longrightarrow (3)$$
$$v = a\omega\frac{\sqrt{a^2 - y^2}}{a}$$
$$v = \omega\sqrt{a^2 - y^2} \longrightarrow (4)$$

Equation (4) gives the velocity and displacement.

Discussions:

(1) At the mean position y=0 velocity is maximum at the mean position from equation (4) :

$$v = \omega\sqrt{a^2 - 0}$$
$$v = a\omega \longrightarrow (5)$$

(2) At the two extreme points y = a

$$v = \omega\sqrt{a^2 - a^2}$$
$$v = 0 \longrightarrow (6)$$

Thus for a particle executing S.H.M the velocity is maximum (aω) at the mean position and zero at the two extreme points.

Acceleration of a particle executing S.H.M

Let us consider a particle executing S.H.M along Y axis.

Let a & T be the amplitude and time period of the given S.H.M

We know that the displacement at any instant of time t is given by

$$y = aSin\omega t \longrightarrow (1)$$

The velocity of the S.H.M at the instant of time t is

$$v = \frac{dy}{dt} = \frac{d(aSin\omega t)}{dt} = a\omega Cos\omega t \longrightarrow (2)$$

If f be the acceleration of the S.H.M at the instant of time t

$$f = \frac{dv}{dt} = \frac{d(a\omega Cost\omega t)}{dt}$$

$$f = -a\omega^2 Sin\omega t \longrightarrow (3)$$

Equation (3) gives the relation between acceleration & time putting equation (1) in (3)

$$f = -\omega^2 y \longrightarrow (4)$$

$$f \propto -y$$

$$acceleration \propto -displacement$$

From equation (4) we find that for a S.H.M the acceleration is always proportional to the displacement.

The negative sign indicates that direction of acceleration is opposite to the direction of displacement.

From equation (4) $\frac{1}{\omega} = \sqrt{\frac{displacement}{acceleration}}$

$$\frac{1}{2\pi/T} = \sqrt{\frac{displacement}{acceleration}}$$

$$T = 2\pi\sqrt{\frac{displacement}{acceleration}} \longrightarrow (5)$$

Equation (5) gives the formula for the time period of a S.H.M