Rotational K.E of a lamina

We can imagine a lamina is made up of large number of point masses say $m_1, m_2, m_3, \ldots$

ω = angular velocity of rotation

r_1, r_2, r_3 … distances of point masses m_1, m_2, m_3, about the axis of rotation respectively.

Let $v_1, v_2, v_3 \ldots$ linear velocity of point masses respectively

$v_1 = r_1 \omega$, $v_2 = r_2 \omega$, $v_3 = r_3 \omega$, ….

∴ K.E of the point masses $\frac{1}{2} m_1 v_1^2$, $\frac{1}{2} m_2 v_2^2$, $\frac{1}{2} m_3 v_3^2$ …..

Since K.E is a scalar quantity their sum would give the total K.E of rotation of the lamina about the given axis

$$E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \ldots$$

$$= \frac{1}{2} m_1 (r_1\omega)^2 + \frac{1}{2} m_2 (r_2\omega)^2 + \frac{1}{2} m_3 (r_3\omega)^2 + \ldots$$

$$= \frac{1}{2} \omega^2 [m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots]$$

$$= \frac{1}{2} \omega^2 \Sigma m r^2 = \frac{1}{2} \omega^2 I$$

Rotational K.E = $\frac{1}{2} I\omega^2$