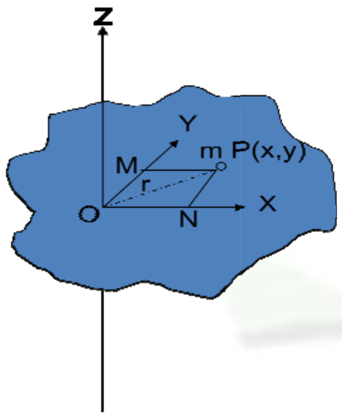
MI– Theorem Of Perpendicular And Parallel Axis

Theorem of Perpendicular Axis

The M.I of a lamina about any perpendicular axis is equal to the sum of moments of inertia of the lamina about two mutually perpendicular axes laying in the plane of the lamina and intersecting each other at the point through which the perpendicular axis passes.



Explanation: Let us consider a lamina given axis OZ is perpendicular to the plane of the lamina. Let us choose two mutually perpendicular axes OX and OY which lie in the plane of the lamina and intersect each other at the point O; through which the perpendicular axis passes.

Let I_x, I_y and I_z be the M.I of the lamina about X, Y and Z axis respectively.

Required to prove   Iz = Ix + Iy

Proof: Let us assume that the lamina is made up of
Let I_x, I_y and I_z be the M.I of the lamina about X, Y and Z axis Respectively.

Required to prove   Iz = Ix + Iy

Proof: Let us assume that the lamina is made up of large number of point masses. Let us consider one such point mass m at P(x, y) at a distance r from O. From P , PN and PM are perpendicular dropped on X and Y axis respectively.
PN = y, PM = x

From right angle triangle ONP $OP^2 = ON^2 + NP^2$

$$r^2 = x^2 + y^2 \longrightarrow (1)$$

M.I of the point mass about X-axis = $m.PN^2 = my^2$

M.I of the whole lamina about X-axis $Ix = \Sigma\, my^2 \longrightarrow (2)$

M.I of the point mass m about Y-axis = $m.PM^2 = mx^2$

M.I of the whole lamina about Y axis $Iy = \Sigma\, mx^2 \longrightarrow (3)$

M.I of the point mass m about Z-axis = $m.OP^2 = mr^2$

M.I of the whole lamina about Z axis $Iz = \Sigma\, mr^2 \longrightarrow (4)$

Multiplying both sides of equation (1) by m and taking Σ

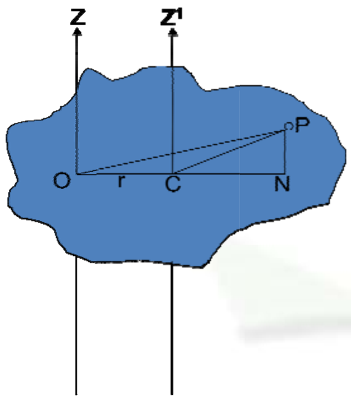
$$\Sigma\, mr^2 = \Sigma\, mx^2 + \Sigma\, my^2 \longrightarrow (5)$$

Putting equation (2) , (3) and (4) in equation (5)

$$Iz = Iy + Ix \qquad \text{Proved}$$

Theorem of parallel axis

The M.I of a lamina about any axis not passing through its centre of mass is equal to the sum of M.I of the lamina about a parallel axis passing through the center of mass and the product of the mass of the lamina and the square of distance between the two parallel axes.



Explanation:

Given M = Mass of the lamina OZ is an axis passing through any point other than its center of mass C

Let us consider an axis CZ^1 passing through the centre of mass and parallel to the given axis OZ.

r = distance between parallel axes

Let I & I_C be the M.I of the lamina about axes OZ and CZ^1 respectively

Required to prove $I = I_C + Mr^2$

Proof: We can imagine the lamina to be made once such point mass m at P. Join OP & CP from P , PN is perpendicular dropped on OC produced.

M.I of the point mass m about the axis OZ = $m.OP^2$

M.I of the whole lamina about axis OZ $I = \Sigma\, m.OP^2$ ⟶ (1)

M.I of the point mass m about the axis $CZ^1 = m.CP^2$

M.I of the whole lamina about the axis $CZ^1 = \Sigma m.CP^2$ ⟶ (2)

From right angled triangle CNP $CP^2 = CN^2 + NP^2$

From right angled triangle ONP $OP^2 = ON^2 + NP^2$ ⟶ (3)

$OP^2 = (OC + CN)^2 + NP^2$

$= OC^2 + 2.OC.CN + CN^2 + NP^2$

Putting equation (3) $OP^2 = CP^2 + OC^2 + 2.OC.CN$ ⟶ (4)

Multiplying both sides of equation (4) by m and taking sigma

$\Sigma\, m.OP^2 = \Sigma\, m.CP^2 + \Sigma\, m.OC^2 + \Sigma\, m.2.OC.CN$ ⟶ (5) [OC = r]

Σ m.OC2 = r2 Σm = M.r2 ⟶ (6) [$M = \Sigma\, m$]

$\Sigma 2m.OC.CN = 2.r\, \Sigma m.CN = 0$ ⟶ (7) [$\Sigma m.CN = 0$]

Putting equation (1), (2), (6) & (7) in equation (5)

$$I = I_c + Mr^2 + 2.r.0 = I_c + M.r^2$$