Composition of two linear S.H.M

Let us consider two S.H.M being executed along the same line say Y axis. The two S.H.M's have same period (T) but have different amplitude and has a phase difference between them.

Let y_1 and y_2 be the displacement of the S.H.M at the instant of time t. Using displacement equation

$$y_1 = aSin\omega t \longrightarrow (1)$$
$$y_2 = bSin(\omega t \pm \phi) \longrightarrow (2)$$

Where a & b are the amplitude

ϕ Is the initial phase difference between these two S.H.M.

When these two S.H.M's are combined let y be the displacement of the resultant motion at the instant of time t

Using the principle of superposition

$$y = y_1 + y_2 \longrightarrow (3)$$

$$y = aSin\omega t + bSin\omega t.Cos\phi \pm bCos\omega t.Sin\phi$$
$$y = Sin\omega t[a + bCos\phi] \pm bCos\omega t.Sin\phi$$
$$y = Sin\omega t.C.Cos\delta \pm Cos\omega t.CSin\delta \longrightarrow (4)$$
$$y = CSin(\omega t \pm \delta) \longrightarrow \mathbf{(7)}$$

Putting
$$a + bCos\phi = C.Cos\delta \longrightarrow (5)$$
$$bSin\phi = C.Sin\delta \longrightarrow (6)$$

C and δ are unknown. Putting Equation (5) and (6) in (4)

From equation (7) which represents a displacement equation we can conclude that the resultant motion is also simple harmonic, having amplitude C and angular frequency ω=2π/T, same as the two given S.H.M and has a phase difference of δ with the first S.H.M.

To find C Squaring and adding equation (5) and equation (6)

$$C^2Sin^2\delta + C^2Cos^2\delta = b^2Sin^2\phi + (a + bCos\phi)^2$$
$$C^2\left[Sin^2\delta + Cos^2\delta\right] = b^2Sin^2\phi + a^2 + b^2Cos^2\phi + 2abCos\phi$$
$$C^2 = a^2 + b^2\left[Sin^2\phi + Cos^2\phi\right] + 2abCos\phi$$
$$C = \sqrt{a^2 + b^2 + 2abCos\phi} \longrightarrow (8)$$

To find δ: dividing equation (5) by equation (6)

$$\frac{CSin\delta}{CCos\delta} = \frac{bSin\phi}{a + bCos\phi}$$
$$\tan\delta = \frac{bSin\phi}{a + bCos\phi} \longrightarrow (9)$$

From equation (8) and (9) the amplitude and phase of the resultant S.H.M can be found.

Special Case: (1) Let the two given S.H.M be in phase.

$$\phi = 0$$
$$\cos\phi = \cos 0 = 1$$
$$\sin\phi = \sin 0 = 0$$

From equation (8) :

$$C = \sqrt{a^2 + b^2 + 2ab.1} = \sqrt{(a+b)^2} = (a+b)$$

The amplitude of the resultant S.H.M is maximum. From equation (9)

$$\tan\delta = \frac{b.0}{a+b} = 0$$
$$\delta = 0$$

The resultant S.H.M is also in phase with the given two S.H.M

Case II : Let two given S.H.M be out of phase

$$\phi = \pi, \cos\phi = \cos\pi = -1$$
$$\sin\phi = \sin\pi = 0$$
$$C = \sqrt{a^2 + b^2 + 2ab(-1)} = \sqrt{(a-b)^2} = (a-b)$$

The amplitude of the resultant S.H.M is minimum From equation (9)

$$\tan\delta = \frac{b.0}{a-b} = 0$$
$$\delta = 0, \pi$$

The resultant S.H.M is in phase with either of the two given S.H.M