Capacitor

Capacitor: Capacitor (or condenser) is a device for storing charge.

It essentially consists of two conducting surfaces such as two plates or two spherical shell or two cylinders etc. kept exactly parallel to each other separated by a very small distance (<0.1mm). The space between the two conductors can be filled with a dielectric medium. One of the two conductors is charged and the other is earthed.

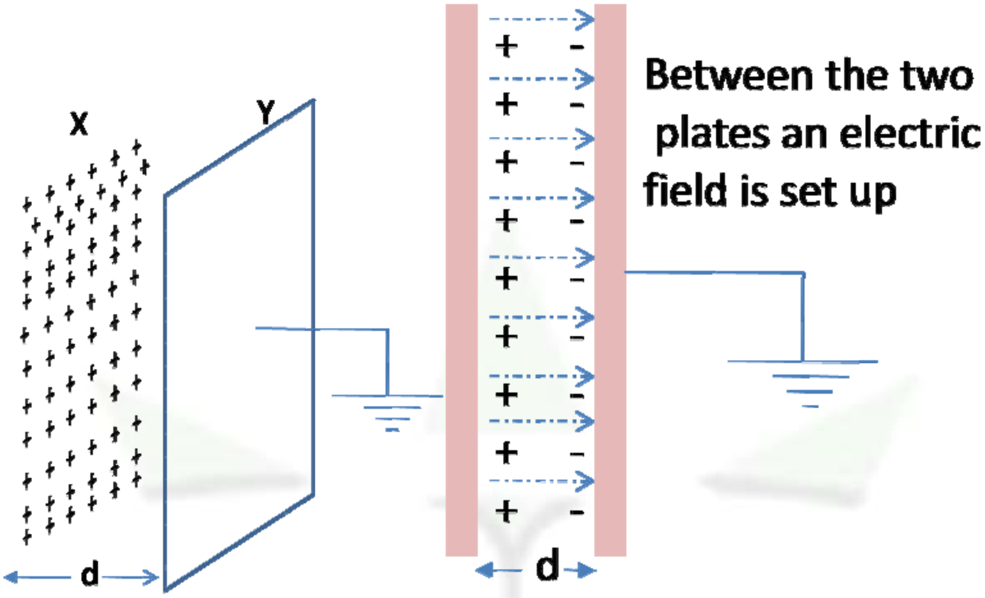
The dielectric between two conductors serves two purposes

(1) It increases the break down potential of the surrounding medium so that more potential can be applied and hence more charge can be stored.

(2) It increases the capacitance of the capacitor.

Explanation: (Break down potential) We know that by applying higher potential a greater charge can be stored in a body. But there is a limit up to which this is true, if we increase the potential beyond the limit the charge will not be stored on the body the surrounding insulating medium becomes conducting medium. As a result the over flowed charge on the body has a tendency to go into the earth hence in that case the excess potential applied goes to the earth by sparking through the new developed conducting medium, i.e. the potential of the surrounding insulating medium breaks down.

(1)Parallel Plate Capacitor:



Given: σ = surface density of charge on the conducting plate X

A = area of the plates

d = the distance of separation between the plates

ε_r = the relative permittivity (dielectric constant) of the medium between the plates

Since gap between the two plates is very very small the field between the two plates is uniform

Let E be the intensity of the electric field in between the two plates.

We know that the intensity at a point near an infinely long charged conducting plate

$$E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \rightarrow (1)$$

Let V be the potential difference betwen the two plates

$$\therefore E = \frac{V}{d}$$

$$V = Ed = \frac{\sigma}{\varepsilon_0 \varepsilon_r} d \rightarrow (2)$$

Charge $Q = \sigma A$

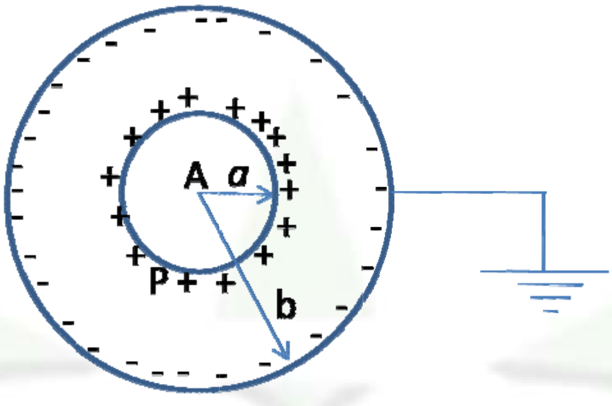
$$\therefore C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0 \varepsilon_r} d} \text{ or } C = \frac{\varepsilon_0 \varepsilon_r A}{d} \rightarrow (3) \text{ or } C = \frac{\varepsilon A}{d} \rightarrow (4)$$

If the space between the plates contains more than one dielectric medium then it is known as compound dielectric. Let t = thickness of dielectric slab (t < d)

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d - \left(1 - \frac{1}{\varepsilon_0}\right) t}$$

(2) Spherical Capacitor:

Case I: The inner shell A is charged and the out shell B is earthed.



a & b = radii of the inner and outer shell respectively.

ε_r = di - electric constant of the medium between the two shells.

Q = charge on the surface of the inner shell A

Charge + Q on A induces bound charge - Q & free charge + Q on the inner and outer surface of the shell B respectively.

The free charge + Q flows to earth.

Consider any point P on A

Potential at P due to the

(i) charge + Q on A, $V_1 = \dfrac{+Q}{4\pi\varepsilon_0\varepsilon_r a}$

(ii) charge - Q on B, $V_2 = \dfrac{-Q}{4\pi\varepsilon_0\varepsilon_r b}$

$\therefore V_P = V_A = V_1 + V_2 = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r}\left[\dfrac{1}{a} - \dfrac{1}{b}\right] = \dfrac{Q(b-a)}{4\pi\varepsilon_0\varepsilon_r ab}$

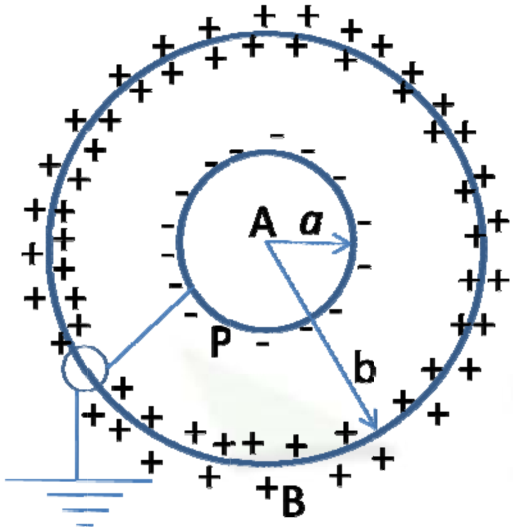
Potential of shell B, $V_B = 0$ it is earthed.

$\therefore$ Potential difference $= V = V_A - V_B = V_A = \dfrac{Q(b-a)}{4\pi\varepsilon_0\varepsilon_r ab}$

The capacitance of the spherical capacitor

$$C = \frac{Q}{V} = \frac{Q}{\dfrac{Q(b-a)}{4\pi\varepsilon_0\varepsilon_r ab}} = \frac{4\pi\varepsilon_0\varepsilon_r ab}{b-a} \rightarrow (1)$$

Case II: The outer shell is charged the inner shell A is earthed



We know that charge always lies on the outer surface of a closed body because it is nearer to the earth (body at zero potential), but in this case since shell B faces earth on its outsider and a body (shell A) at zero potential on its inner side hence the charge on B will be distributed on its outer surface as well as inner surface.

Let Q_1 & Q_2 charge on the outer and inner surface of the shell B respectively.

$Q = Q_1 + Q_2 \rightarrow (1)$

The charge $+Q_2$ on the inner surface of B indicates $-Q_2$ on A.

Consider any point P on A

Potential at P due to the

(i) $-Q_2$ on A, $V_1' = \dfrac{-Q_2}{4\pi\varepsilon_0\varepsilon_r a}$

(ii) $+Q_2$ on B, $V_2' = \dfrac{+Q_2}{4\pi\varepsilon_0\varepsilon_r b}$

(iii) $+Q_1$ on B, $V_3' = \dfrac{+Q_1}{4\pi\varepsilon_0\varepsilon_r b}$

$$\therefore V_A = V_1' + V_2' + V_3' = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r b} - \frac{Q_2}{4\pi\varepsilon_0\varepsilon_r}\left[\frac{1}{a} - \frac{1}{b}\right] = 0$$

$V_A = 0$ because A is earthed.

$$\frac{Q_1}{4\pi\varepsilon_0\varepsilon_r b} = \frac{Q_2}{4\pi\varepsilon_0\varepsilon_r}\frac{(b-a)}{ab}$$

$$\therefore Q_2 = \frac{Q_1}{b-a} \rightarrow (3)$$

Putting equation (3) in equation (2)

$$Q_1 + \frac{Q_1 a}{b-a} = Q$$

$$Q_1\left[\frac{b-a+a}{b-a}\right] = Q$$

$$Q_1 = \frac{Q(b-a)}{b} \rightarrow (4)$$

Potential at any point on the surface of the shell B due to

(i) Charge $-Q_2$ on A, $V_1 = \frac{-Q_2}{4\pi\varepsilon_0\varepsilon_r b}$

(ii) Charge $+Q_2$ on B, $V_2 = \frac{+Q_2}{4\pi\varepsilon_0\varepsilon_r b}$

(iii) Charge $+Q_1$ on B, $V_3 = \frac{+Q_1}{4\pi\varepsilon_0\varepsilon_r b}$

Total potential of the shell B: $V_B = V_1 + V_2 + V_3$

$$V_B = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r b}$$

Potential difference between the shells $V = V_B - V_A = V_B - 0$

$$V = V_B = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r b} \rightarrow (5)$$

Putting equation (4) in equation (5):

$$V = \frac{Q(b-a)}{b.4\pi\varepsilon_0\varepsilon_r b}$$

$\therefore$ Capacitance of the capacitor: $C = \frac{Q}{V} = \frac{Q}{\frac{Q(b-a)}{4\pi\varepsilon_0\varepsilon_r b^2}} = \frac{4\pi\varepsilon_0\varepsilon_r b^2}{(b-a)}$

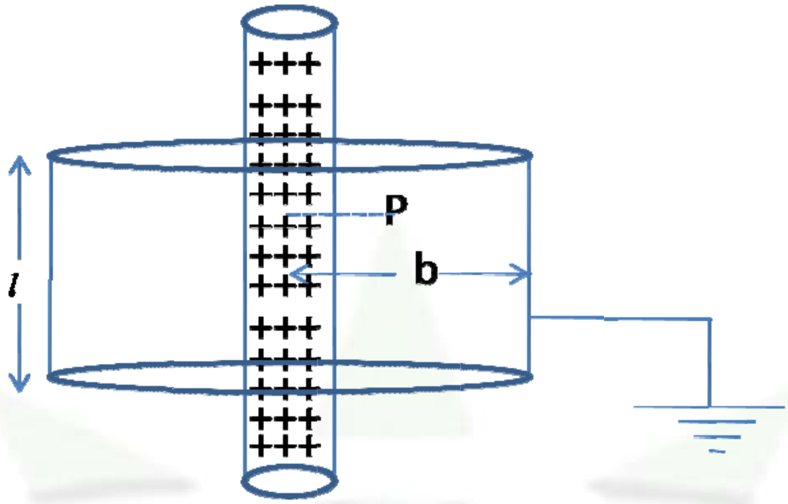
If C_o & C_i be the capacitance of spherical capacitor when outer shell is earthed and inner shell is earthed then

$$C_i = \frac{4\pi\varepsilon_0\varepsilon_r ab}{(b-a)}$$

$$C_o = \frac{4\pi\varepsilon_0\varepsilon_r bb}{(b-a)}$$

$$C_o > C_i$$

Capacitance of a cylindrical capacitor:



a and b = radii of inner and outer cylinder respectively

λ = charge per unit length

l = length of the cylinder

ε_r = relative permittivity of the medium between the two cylinders

consider any point P at a distance r from the axis. From the application of Gauss's theorem we know that intensity at P due to the charged inner cylinder

$$E = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r r}$$

$$\text{But } E = -\frac{dv}{dr}$$

$$dv = -Edr$$

Total potential difference between the two cylinders

$$v = \int dv = \int Edr = \int_a^b \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r r} dr$$

$$v = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r} \log_e \frac{b}{a}$$

$$\therefore C = \frac{Q}{V} = \frac{\lambda l}{\frac{\lambda}{2\pi\varepsilon_0\varepsilon_r} \log_e \frac{b}{a}}$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{\log_e \frac{b}{a}}$$