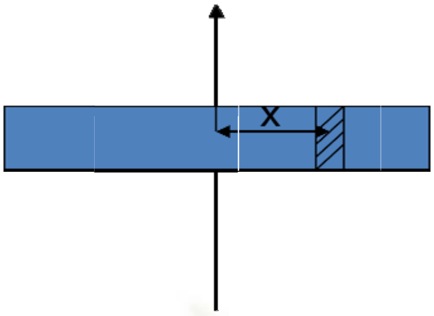
M.I of a thin rod

(a) About an axis passing through the center and perpendicular to the length of the rod



AB is the given rod and OZ is the given axis.

M = Mass of the rod

L = length of the rod

Mass per unit length = M/L

Let us imagine an element of the rod of length dx at a C at a distance x from the axis.

Mass of the element = $\frac{M}{L}dx$

Since the element can be treated as a point mass the M.I of the lamina about the given axis

= mass of the element x (distance) ^{2}

$$= \frac{M}{L}dx.x^2$$

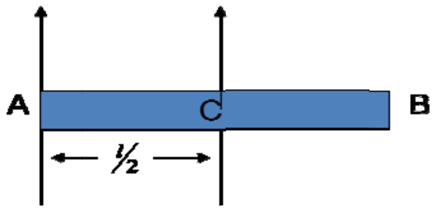
$$= \frac{M}{L}x^2 dx$$

Since the rod can be assumed to be made up of many such elementary strips, M.I of the rod about the given axis can be obtained by integrating equation (1) between the limits x = 0 to L/2 and multiplying by 2

$$I = 2\int_0^{l/2} \frac{M}{l}x^2 dx = 2\frac{M}{l}\int_0^{l/2} x^2 dx$$

$$= 2\frac{M}{l}\left[\frac{x^3}{3}\right]_0^{l/2} = \frac{Ml^2}{12}$$

(b) About an axis passing through one end of the rod and perpendicular to its length



Applying theorem of parallel axis $$I = Ic + M\left(l/2\right)^2 = \frac{Ml^2}{12} + \frac{Ml^2}{4} = \frac{Ml^2}{3}$$