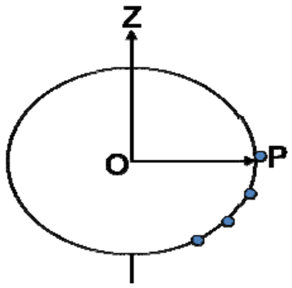
M.I of a circular ring

(A) About an axis passing through the center



Given M = Mass of the ring

R = Radius of the ring

OZ is the given axis

We imagine the ring is made up of large number of point masses. Let us consider one such point mass m at P

M.I of the point mass about the given axis = mR^2

M.I of the whole lamina about the given axis $= \Sigma mR^2 = MR^2$

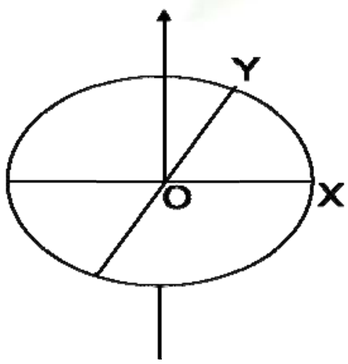
(B) About any diameter of the ring

Let us choose two mutually perpendicular diameters.

Let Ix and Iy be the M.I about XX1 and YY1 respectively.

Applying theorem of perpendicular axis $I_z = I_x + I_y$

[M.I about any diameter is same i.e. $I_x = I_y = I$]



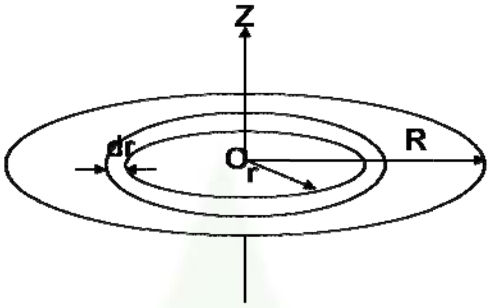
$MR^2 = I + I$

$2I = MR^2$

$$I = \frac{MR^2}{2}$$

Moment of Inertia of a circular disc (circular lamina)

(A) About an axis passing through the centre and perpendicular to the plane of the disc



Given: M = Mass of the disc
R = Radius of the disc

Mass per unit area $= \frac{M}{\pi R^2}$

Let us imagine an elementary circular strip of radius r and radial thickness δr

Area of the elementary circular strip = circumference x thickness $= 2\pi r \times \delta r$

Mass of the elementary strip $= \frac{M}{\pi R^2} \times 2\pi r \times \delta r = \frac{2M}{R^2} r\delta r$

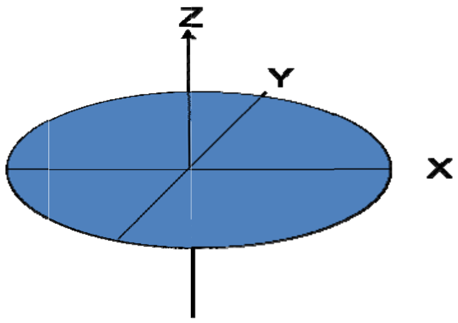
Since the given axis OZ passes through the center of the elementary ring and is perpendicular to the plane of the ring, M.I of this elementary ring about the given axis = Mass of the ring X (radius)^2

$$= \frac{2M}{R^2} r\delta r \times r^2 = \frac{2M}{R^2} r^3 \delta r \longrightarrow (1)$$

Since the disc can be assumed to be made up of many such elementary strips. M.I of the disc about the given axis can be obtained by integrating equation (1)

$$Iz = \int_0^R \frac{2M}{R^2} r^3 dr = \frac{2M}{R^2}\left[\frac{r^4}{4}\right]_0^R = \frac{MR^2}{2} \longrightarrow (2)$$

(B) M.I of the disc about its diameter



Applying theorem of perpendicular axis

Iz = Ix + Iy

$$\frac{MR^2}{2} = I + I$$

$$I = \frac{MR^2}{4}$$