Compound Pendulum:

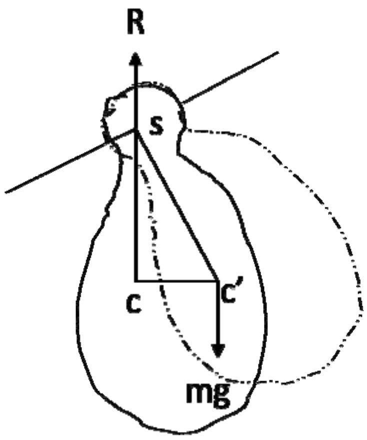
Definition: It is just a rigid body capable of rotating freely about a horizontal axis passing through any point other than the centre of mass.

M = mass of the body

I = M.I of the body about the axis of suspension

I_o = M.I of the body about an axis passing through the center of mass 'c' and parallel to the given axis of suspension.

From the theorem of parallel axis $I = I_o + ml^2 \longrightarrow$ (1)



Where = distance SC = distance between the point of suspension and center of mass

Force acting on the body

(1) Gravitational force mg along vertically downward direction

(2) The reaction R of the axis at the point of suspension S along vertically upward direction

$$\sum F = 0$$

$$R - mg = 0$$
$$R = mg$$

The forces R & mg must be collinear i.e in the position of equilibrium the point c must be vertically below S.

Let the body be given a small angular displacement θ c goes to c', the forces R & mg although are equal in magnitude become non collinear and since they are parallel and opposite they constitute a couple.

Moment of couple $= mg \times c'c = mgl \sin\theta$

Direction of the couple which is clockwise in the figure restores the body back to its position and hence for this restoring nature it is given a negative sign.

Restoring couple $= -mgl\sin\theta = -mgl\theta \longrightarrow$ (2)

θ being small $\sin\theta \approx \theta$

The couple acting on the body produces an angular acceleration, using Newton's 2nd law

I X angular acceleration = restoring couple

$= -mgl.\theta$

Angular acceleration $= -\dfrac{mgl}{I}$. angular displacement $\longrightarrow$ (3)

Since I, m, l are all constant from equation (3) we get

Angular acceleration $\propto$ - angular displacement

Hence the motion of the body is S.H.M

Let T = Time period of S.H.M

$$T = 2\pi\sqrt{\frac{displacement}{acceleration}} \longrightarrow \quad (4)$$

$$T = 2\pi\sqrt{\frac{I}{mgl}} \longrightarrow \quad (5)$$

$$T = 2\pi\sqrt{\frac{I_0 + ml^2}{mgl}}$$

$$T = 2\pi\sqrt{\frac{mk^2 + ml^2}{mgl}} = 2\pi\sqrt{\frac{l + \frac{k^2}{l}}{g}} \longrightarrow \quad (6)$$

Put $\dfrac{k^2}{l} = l'$ $\quad T = 2\pi\sqrt{\dfrac{l + l'}{g}} \longrightarrow$ (7)

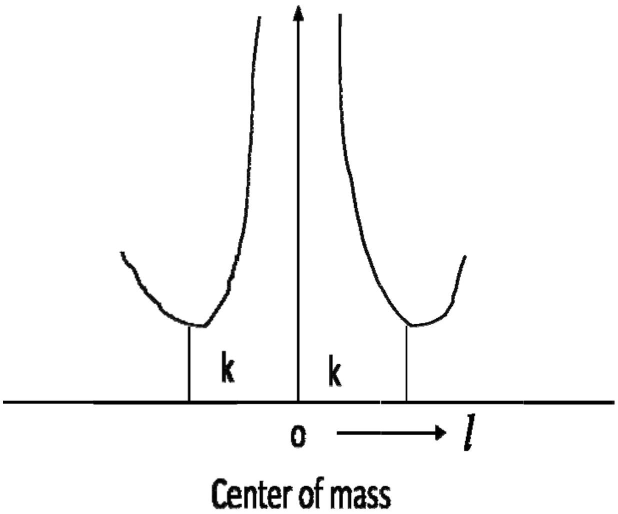
$l + l' = L$ $\quad T = 2\pi\sqrt{\dfrac{L}{g}} \longrightarrow$ (8)

Equation (8) resembles the expression for time period of a simple pendulum hence the length L is known as the equivalent length of a simple pendulum. If we want to construct a simple pendulum having a time period same as the given compound pendulum the length of the simple pendulum should be L= $l + l'$

When $l = \pm k$ T is minimum, when l=0, T=∞

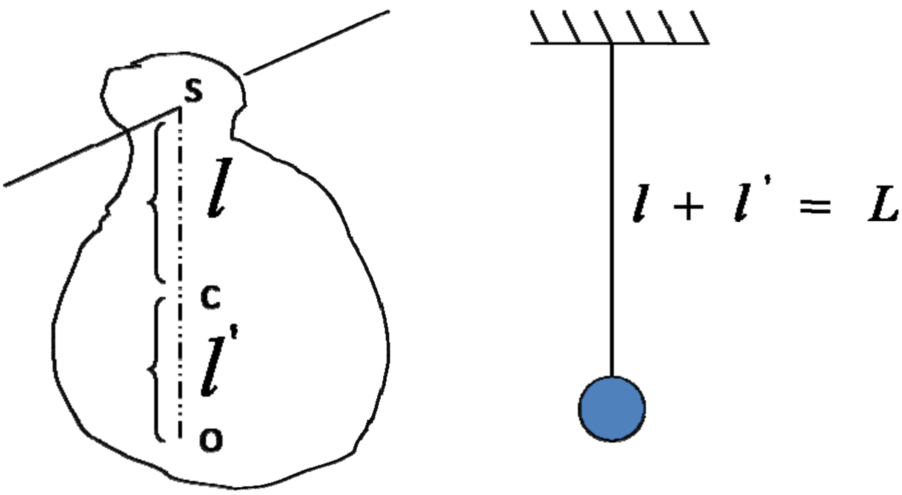
When l = ∞ T = ∞

There are four points of suspension collinear with centre of mass about which the time period of Oscillation is same.



Discussions:

(1) Center of suspension and center of oscillation



os = the point through which the axis passes is known as center of suspension.

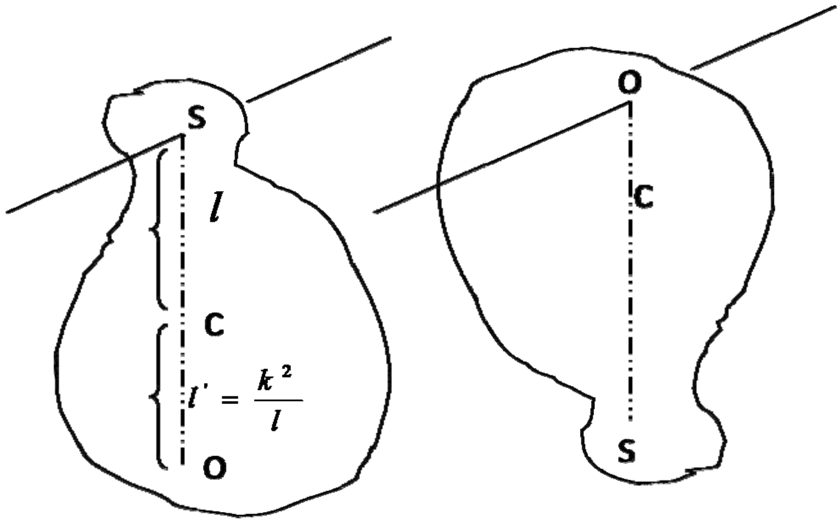
$$l' = \frac{k}{l}$$

sc= *l* , Joining sc and producing up to co .

The point o is known as center of oscillation corresponding to the center of suspension s.

Since center of suspension is not a fixed point similarly center of oscillation is also not a fixed point & for every center of suspension there is one center of oscillation.

(2) It can be shown that the center of suspension and center of oscillation are interchangeable i.e time period about a center of suspension & corresponding center of oscillation are same.

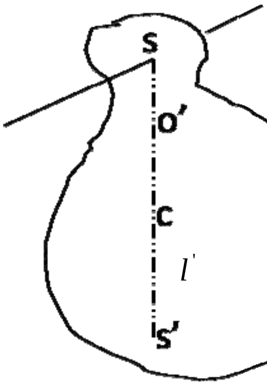


$$T_s = 2\pi\sqrt{\frac{l+\frac{k^2}{l}}{g}} = 2\pi\sqrt{\frac{l+l'}{g}}$$

$$T_o = 2\pi\sqrt{\frac{l'+\frac{k^2}{l'}}{g}} = 2\pi\sqrt{\frac{l'+l}{g}}$$

$$l' = \frac{k^2}{l} \therefore l = \frac{k^2}{l'}$$

$T_s = T_o$



(3) There are four points collinear with CG about which time period are same

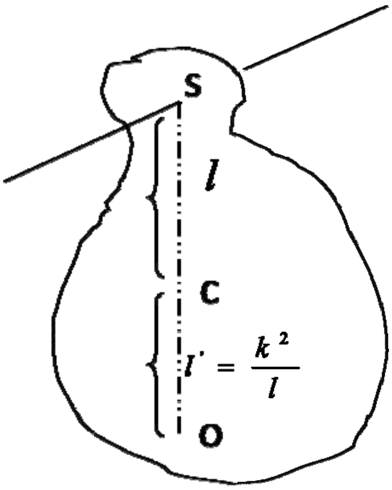
cs = l = cs', co = = co'

$$T_s = T_{s'} = 2\pi\sqrt{\frac{l + l'}{g}}$$

$$T_o = T_{o'} = 2\pi\sqrt{\frac{l' + \frac{k^2}{l'}}{g}}$$

$$T_o = T_{o'} = 2\pi\sqrt{\frac{l' + l}{g}}$$

(4)



$$T^2 = \frac{4\pi^2}{g}\left[l + \frac{k^2}{l}\right]$$

When $l = 0$, T= ∞

When $l = \infty$, T= ∞