



Mathematics

- (a) Verify: $A \Delta (B \Delta C) = (A \Delta B) \Delta C$ for sets
 $A = \{x \mid |x| < 2\}$, $B = \{x \mid |x-1| < 2\}$ and $C = \{x \mid |x+1| < 2\}$

(b) Show that for all real values of x , the expression $\frac{x^2 + 7x + 7}{x^2 + x + 1}$ cannot take any value outside the interval $[-1, 7]$

(c) A candidate is required to answer 7 questions out of 12 questions which are divided into two groups of 6 questions each. He is not permitted to attempt more than 5 questions from either group. In how many different ways can he choose the seven questions?
- Evaluate : (i) $\frac{d}{dx} [\sin(\log x) - \log(\sin x)]$

(ii) $\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$

(b) Determine a and b so that the functions f given by

$$\begin{aligned} f(x) &= \frac{1-\sin^2 x}{\cos^2 x} && \text{if } x < \frac{\pi}{2} \\ &= a && \text{if } x = \frac{\pi}{2} \\ &= \frac{b(1-\sin x)}{(\pi-2x)^2} && \text{if } x > \frac{\pi}{2} \end{aligned}$$

is continuous at $x = \frac{\pi}{2}$
- (a) Prove that the circle circumscribing the triangle formed by any three tangents to a parabola passes through the focus.

(b) A point P moves such that the tangents PT_1 and PT_2 from it to the hyperbola $4x^2 - 9y^2 = 36$ are mutually of right angles. Find the equation of the locus of P.
- (a) The probability that A narrates an incident truthfully is $\frac{3}{4}$ and for B the probability of truthful narration is $\frac{4}{5}$. What is the probability that they will contradict each other in narrating the same incident?

(b) Let the frequency mean and standard deviation be denoted by $n_1 \bar{x}_1$ and S_1 for one set of variates and $n_2 \bar{x}_2$ and S_2 for the second set. Show that the variance S^2 of the composite set is given by $NS^2 = n_1 S_1^2 + n_2 S_2^2 + n_1 d_1^2 + n_2 d_2^2$
Where $N = n_1 + n_2$ and $d_1 = \bar{x}_1 - \bar{x}$
- (a) An observer at an Anti Aircraft post. A identifies an enemy aircraft due east of his post at an angle of elevation 60° . At the same instant detection post D situated 4 km due south of A reports that the aircraft is at an angle of elevation 30° . Calculate the altitude at which the plane is flying.

(b) Find the greatest angle in the triangle, the sides of which are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$.
- (a) An aeroplane travelling at the rate of 60 miles per hour horizontally a height of $2\frac{8}{11}$ miles from the ground has to drop a bomb on a certain target on the ground. At what



Mathematics

horizontal distance from the target must the bomb be dropped so that it may exactly be hits the target?

(b) From a balloon rising vertically with a constant acceleration f a stone is thrown up with a velocity v relative to the balloon overtakes the stone after t seconds, show that $v = \frac{t}{2}(f + g)$.

7. (a) If $x = a + b$, $y = a\omega + b\omega^2$ and $z = a\omega^2 + b\omega$ where ω is the complex cube root of unity, then $x^3 + y^3 + z^3 = \dots$

(b) if $(1 + x)^n = \sum_{r=0}^n C_r x^r$ then $(1 + \frac{C_1}{C_0})(1 + \frac{C_2}{C_1}) \dots \dots + (1 + \frac{C_n}{C_{n-1}}) = \dots$

8. (a) $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{\cot x - \cot a} =$

(b) $\lim_{h \rightarrow 0} \frac{2[\sqrt{3} \sin(\frac{\pi}{6} + h) - \cos(\frac{\pi}{6} + h)]}{\sqrt{3}h(\sqrt{3} \cosh - \sinh)}$

(c) $\int_0^{x^3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} - \sqrt{\cos x}} =$

9. (a) The line $3x + 5y = k$ touches the ellipse $16x^2 + 25y^2 = 400$ if $k =$

(b) The equation $16x^2 - 24xy + 9y^2 + 28x + 14y + 21 = 0$ represents

10. (a) $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \dots$

(b) The planes $\vec{r} (7\vec{j} - 5\vec{k})$, $\vec{r} (2\vec{i} + 5\vec{j} + 3\vec{k}) = 0$, $\vec{r} (\vec{i} - \vec{j} - 4\vec{k}) = 2$

(i) meet in a point

(ii) Intersect in a line

(iii) intersect in mutually parallel lines

(iv) are parallel.

(b) A body of weight 30 lbs in equilibrium on a smooth plane inclined at an angle of 45° to the horizontal and is supported by a force P acting at an angle θ with the plane. If R is the reaction of the plane, then $R/P = \dots$