



Mathematics

1. (a) If A, B, C are three sets such that $A \Delta B + A \Delta C$ then prove that $B = C$
(b) If $(1 + x)^n = \sum_{r=0}^n C_r x^r$ then prove that

$$\sum_{r=0}^n \frac{C_r}{(r+1)2^{r+1}} = \frac{3^{n+1} - 2^{n+1}}{(n+1)2^{n+1}}$$

2. (a) By defining $f\left(\frac{\pi}{4}\right)$ suitably, obtain a continuous extension of the function f given by

$$f(x) = \frac{\sqrt{2} \cos x - 1}{\cos x - 1}, x \neq \frac{\pi}{4}$$

- (b) a particle moving in a straight line passes vertically above two points A and B on a horizontal plane and 100 metres apart. When above A its angle of elevation as seen from B is 60° and when above B, its angle of elevation as seen from A is 30° . At what distance from the point A will the particle touch the ground?
3. (a) Evaluate $\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$
(b) If P is a point inside the triangle ABC such that $\angle PAB = \angle PBC = \angle PCA = \alpha$.
Show that $\cot \alpha = \cot A + \cot B + \cot C$

4. (a) Evaluate $\int_0^{\pi/2} \frac{dx}{5+4 \cos x}$

(b) A tangent to the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = 1$ cuts the co-ordinate axes in A and B. Show that the equation of the locus of the mid-point of AB is $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$

5. (a) The following table gives the frequency distribution of marks obtained by a batch of 20 students.

Marks	5	15	25	35	45
No. of students	5	4	6	3	2

Calculate the Mean and Standard deviation.

(b) If the normal's at four point $P_i(x_i + y_i)$ $i = 1, 2, 3, 4$ on the rectangular hyperbola $xy = c^2$ meet in the point $Q(h, k)$ prove that

(i) $x_1 + x_2 + x_3 + x_4 = h$

(ii) $y_1 + y_2 + y_3 + y_4 = k$

(iii) $x_1 x_2 x_3 x_4 = y_1 y_2 y_3 y_4 = -c^4$

6. (a) A particle which starts from rest moves at first with a constant acceleration a and then with a constant retardation b. If after moving for a time t from the start the particle comes to rest at a distance from the starting point, prove that $2(a + b)s = abt^2$



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- (b) A heavy uniform rod having a mass of 40 lbs is suspended in a horizontal position by vertical strings from its two ends each of which can sustain a tension of 35 lbs wt. How far from the centre of the rod should a body of mass 20lbs be attached to the rod so that one of the strings may just break?
7. (a) If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then bc^2, ca^2, ab^2 are in AP, GP or HP?
- (b) The area of the triangle on the Argand diagram formed by the complex numbers z, iz and $z+iz$ is
8. (a) In a triangle ABC if $a = \sqrt{3} + 1$ cms, $b = \sqrt{3} - 1$ and $\angle C = 60^\circ$ then the smallest angle in the triangle is
- (b) If $A(-1,2,3)$, $C(1,1,1)$ and $C(2,-1,3)$ are points on a plane, a unit normal vector to the plane ABC is
- (c) if the vectors $a\vec{i} + \vec{j} + \vec{k}$, $a\vec{i} + b\vec{j} + \vec{k}$ and $a\vec{i} + \vec{j} + c\vec{k}$ ($a = b = c \neq 1$) are coplanar then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = \dots$
9. (a) If R is the resultant of two forces P and Q and If $\frac{P}{3} = \frac{Q}{7} = \frac{R}{5}$ then the angle between P and R is
- (b) If $f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ then $f(\pi/a) = \dots$
10. (a) The normal at the point $P(at_1^2, 2at_1^3)$ then $t_3 = \dots$
- (b) The angle of projection of a particle when its range on a horizontal plane is $4\sqrt{3}$ times the height attained is given by
- (c) A bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from a randomly chosen bag and is found to be red. The probability that it was drawn from bag B was