



Mathematics

1. (a) Prove that $\left| \vec{a} + \vec{b} \right|^2 + \left| \vec{a} - \vec{b} \right|^2 = 2\left| \vec{a} \right|^2 + 2\left| \vec{b} \right|^2$
(b) The elevation of the top of a tower at a point E. due east of the tower is α , and at the point S due south of the tower is β . Prove that it's elevation at a point midway between E and S is given by
 $(\tan^2 \alpha + \tan^2 \beta)^2 = \tan \alpha \tan \beta$

2. (a) If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$ then show that
 $C_1 - 2C_2 + 3C_3 - \dots + (-1)^{n-1}C_n = 0$
(b) From 7 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways this can be done so as to include at least one lady.

3. (a) Evaluate :

$$\int_0^3 \frac{dx}{(x+2)\sqrt{x}-1}$$

- (b) If $f(x) = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)f'(x) - xf(x) = 1$
4. (a) Find the coordinates of the focus of the parabola
 $y^2 - 4y - 6x + 13 = 0$
(b) Prove that the line $y = x + \frac{\sqrt{5}}{6}$ touches the ellipse $2x^2 + 3y^2 = 1$
5. (a) One end of a heavy uniform rod of weight W rests on a smooth horizontal plane and a string tied to the other end is fastened to a fixed point above the plane. Find the tension of the string.
(b) Two rough particles connected by a tight string rest on an inclined plane. If their weight's and coefficients of friction be w_1, w_2 and μ_1, μ_2 respectively, then show that the greatest inclination of the plane for equilibrium is $\tan^{-1} \left[\frac{\mu_1 w_1 + \mu_2 w_2}{w_1 + w_2} \right]$
6. (a) To a man walking at a rate of 3 km/hr, the rain appears to fall vertically. Find the actual direction of the rain if its relative velocity $3\sqrt{3}$ km/hr.
(b) Two stone is thrown horizontally with $\sqrt{2gh}$ from the top of the tower of height h . Find where it will strike the level ground through the foot of the tower.
7. (a) The maximum value of $y = \sin x + \cos x$ is ..
(b) If a is a positive integer then the value of
$$\lim_{x \rightarrow \infty} \frac{x^n + a_1x^{n-1} + \dots + a_n}{x^n + b_1x^{n-1} + \dots + b_n}$$

(c) The angle A of the ΔABC in which $(a+b+c)(b+c-a) = 3bc$ is ...
(d) If P & Q are subsets of a set x, then $(P \cap Q)^c$ is equal to ...



Mathematics

8. (a) If ω is a complex cube root of unity, then $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)$ is equal to ...
(b) The sum of the infinite series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots \infty$ is ...
9. (a) $\int_0^{\pi/2} (a \cos^2 x \times b \sin^2 x) dx = \dots$
(b) In the argand plane of the vertices of a triangle are z_1, z_2, z_3 the centroid of the triangle is ...
10. (a) The mean of 5 observations is 3 and the variance is 2. If the three of the five observations are 1,3,5 then the two are ...
(b) The probability that India wins a cricket test match against England is $(1/3)$. If India and England play three matches, the probability that India will win at least one test match is