



## Mathematics

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- ( a ) If  $Z_1, Z_2, Z_3$  are the vertices of an equilateral triangle, in the Argand plane, then show that  
 $Z_1^2 + Z_2^2 + Z_3^2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$

( b ) Find the real  $x$ , satisfying  $|x^2 - x - 6| + x + 2 = 0$
- ( a ) If in a triangle  $ABC$ ,  $\tan A/2, \tan B/2, \tan C/2$  are in H.P then show that sides  $a, b, c$  are in AP.

( b ) The angles of elevation of the top of a tower at the top and the foot of a pole of height 10m are  $30^\circ$  and  $60^\circ$  respectively. Find the height of the tower.
- ( a )  $f$  is a real valued function defined on the real's and  $(x + y) = f(x) + f(y)$  for all reals  $x, y$ . If  $f$  is continuous at 'O' then show that  $f$  is everywhere continuous on reals.

( b ) Evaluate the following :

$$\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$$
- ( a ) If the normal at  $t_1$  on the parabola  $y^2=4x$  meets it again at  $t_2$  find the relation between  $t_1$  and  $t_2$

( b ) Prove that the locus of the middle point of the portion of a tangent to  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  included between the axes is  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$
- ( a ) The length of an inclined plane is 5 feet and the height is 3 feet a force of 3 lb. weight acting parallel to the plane will just prevent a weight 10 lb. from sliding down. Show that the co-efficient of friction is  $3/8$

( b ) A man can throw a ball 60 ft. vertically upwards. Find the greatest horizontal distance he can throw it.
- ( a ) If  $\vec{a}, \vec{b}$  are vectors such that  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$  then show that  $a, b$  are perpendicular.

( b ) A bag contains 'a' white and 'b' black balls. Two players A and B alternately draw a ball from the bag, replacing the ball each time after the draw till one of them draws a white ball and wins the game. A begins the game. If the probability of A winning the game is three times that of B, then show that  $a:b = 2:1$
- ( a ) If  $Z_r = \cos \frac{\pi}{2r} + i \sin \frac{\pi}{2r}, r = 1, 2, \dots$  then  $Z_1, Z_2, \dots$  to  $\infty$  is ...

( b ) If the sum of the roots of the quadratic equation  $ax^2 + bx + c = 0$  is equal to the sum of the squares of their reciprocals then  $b^2/ac + bc/a^2 = \dots$



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8. (a) Derivative of  $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$  w.r.t  $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$  is ...  
(b)  $\int_{-2}^1 |1-x^2| dx =$   
(c) If  $\vec{a}, \vec{b}, \vec{c}$  are three non-coplanar vectors and  $\vec{p}, \vec{q}, \vec{r}$  are the vectors where  
 $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a} \ \vec{b} \ \vec{c}]}$  then  $(\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r} = \dots$
9. (a) The point of contact of line  $y = x-1$  with  $3x^2 - 4y^2 = 12$  is ...  
(b) The foci of  $16x^2 + 25y^2 = 400$  are ...
10. (a) The range of random variable  $x$  is  $\{1, 2, 3 \dots\}$ . The probability that  $x$  takes the value  $k$  is  $\frac{\lambda^k}{k!}$  ( $k=1,2,\dots$ ). Then  $\lambda = \dots$   
(b) The standard deviation of  $a, a+d, a+2d, \dots, a+2^{nd}$  is ...