



## Mathematics

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- (a) Let  $z = x + iy$ , where  $x, y$  are variables. Prove that if  $2|z-1| = |z-2|$  then  $3(x^2+y^2) = 4x$ .

(b) Find the three cube roots of unity.
- (a) If  $\alpha$  and  $\beta$  are the roots of  $5x^2 - 20x + 12 = 0$  then the value of  $\alpha^3 + \beta^3$  is  $35\frac{1}{5}$

(b) show that the function  $\frac{x^2-x+1}{x-1}$  cannot take any value between -1 and 3 for any real value of  $x$
- (a) Show that  $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$  for  $-\infty < x < \infty$

(b) When the elevation of the sun is  $30^\circ$ , show that the length of the shadow cast by a tower of 150 ft height is  $150\sqrt{3}$  ft.
- (a) Prove that  $\lim_{x \rightarrow \infty} \frac{2x^3 - 4x + 7}{3x^3 + 5x^2 - 4} = \frac{2}{3}$

(b) Show that  $f(x) = \frac{e^x - 1}{e^x + 1}$  when  $x \neq 0$ ,  $f(0) = 0$  discontinuous at  $x = 0$
- (a) If  $x^y = e^{x-y}$ , show that  $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

(b) If  $ax^2 + 2hxy + by = 1$ , prove that  $\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^2}$
- (a) Evaluate  $\int x \log(1+x) dx$

(b) Prove that  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$
- (a) Prove that necessary and sufficient condition that three non-parallel and non-null vectors be coplanar is  $[\vec{a}, \vec{b}, \vec{c}] = 0$

(b) If the tangents at two points P and Q on a parabola meet in T and S is the focus, prove that  $ST^2 = SP \cdot SQ$
- (a) Prove that if three coplanar parallel forces be in equilibrium, each is proportional to the distance between the other two.

(b) Show that the probability of getting head in a throw of a coin is  $\frac{1}{2}$ .
- (a) The equation of ellipse in the form  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  given the eccentricity to be  $\frac{2}{3}$  and latus rectum  $\frac{2}{3}$  is ....

(b) The probability that a company executive will travel by train  $\frac{2}{3}$  and that he will travel by is  $\frac{1}{5}$ . The probability of his travelling by train or plane is ...

(c) The hyperbola is the conic with eccentricity ....



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10. ( a ) The resultant of two velocities  $u$  and  $au$  is  $bu$  and is at right angles to  $u$ . The  $a^2 - b^2 = ..$
- ( b ) A relation  $f$  such that just one  $y$  corresponds to each  $x$  in the domain  $X$  is called a ...
- ( c )  $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{c} \cdot \vec{a}) \cdot \vec{b} - (\vec{c} \cdot \vec{b}) \cdot \vec{a}$  is known as the ..... Triple product of three vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$ .