



Mathematics

1. If ω is cube root of unity, show that
 - (a) $(1-\omega-\omega^2)^5 + (1+\omega-\omega^2)^5 = 32$
 - (b) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ to 100 factors is equal to 1
2. (a) Solve : $2x - 1 = 3 + \frac{2}{x-2}$
 - (b) If $(b-c)x^2 + (c-a)x + (a-b) = 0$ has equal roots show that $2b = (a+c)$
 - (c) If α, β are the roots of the equation $x^2 - p(x+1) - c = 0$ show that $(\alpha+1)(\beta+1) = 1-c$
3. (a) Show that $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
 - (b) From the top of a cliff 300 metres high the top of a tower was observed at an angle of depression 30° and from the foot of the tower the top of the cliff was observed at an angle of 45° . Find the height of the tower and also the distance between the tower and the cliff.
4. (a) R is the set of real numbers and $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined as $f(x) = 3x^2+2$ and $g(x) = 3x-1$ for all $x \in R$. Find the composite function $f \circ g$ and $g \circ f$ from $R \rightarrow R$
 - (b) $f: R \rightarrow R$ is defined by $f(x) = x^2$ for all $x \in R$. Given $\epsilon > 0$ find a $\delta > 0$ to show that f is continuous at $x=2$
5. (a) if $y = \sqrt{x \log_e x}$ and $\frac{dy}{dx}$ at $x=e$
 - (b) if $x^3 = e^{x-y}$ determine $\frac{dy}{dx}$ in terms of x
6. (a) Integrate $\frac{x}{x^4-x^2+1}$ with respect to x
 - (b) Evaluate $\int_0^{1/\sqrt{2}} \frac{\sin^{-1}x}{(1-x^2)\sqrt{1-x^2}} dx$
7. (a) Find the unit vector perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} - \hat{k}$
 - (b) For any vector \vec{a} show that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k})$ is equal to $2\vec{a}$
8. (a) Show that $3x^2 + 4y^2 - 6x + 8y - 5 = 0$ represents an ellipse and determine its centre, foci and eccentricity.
 - (b) Find the equations of the tangent and normal at $(a, -2a)$ to the parabola $y^2 = 4ax$



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9. (a) The resultant of two forces P and Q acting at a point at angle α is $(2m+1)\sqrt{(P^2 + Q^2)}$ and when the angle between them is made $\frac{\pi}{2} - \alpha$ the resultant becomes $(2m-1)\sqrt{(P^2 + Q^2)}$

(b) A particle is projected vertically upwards with a velocity u m/sec and after t seconds another particle is projected upwards from the same point and with the same velocity. Find the time for the two particles to meet.

10. (a) A and B are two candidates seeking admission into course. The probability that A is selected is 0.6 and the probability that both A and B are selected is at most 0.3 is it possible that the probability of B getting selected is $\frac{3}{4}$? Why?

(b) Find the variance of the distribution of 300 telephone calls according to their duration in seconds given below :

Duration (in seconds)	0-30	30-60	60-90	90-120	120-150	150-180
No.of calls	13	21	47	86	35	48