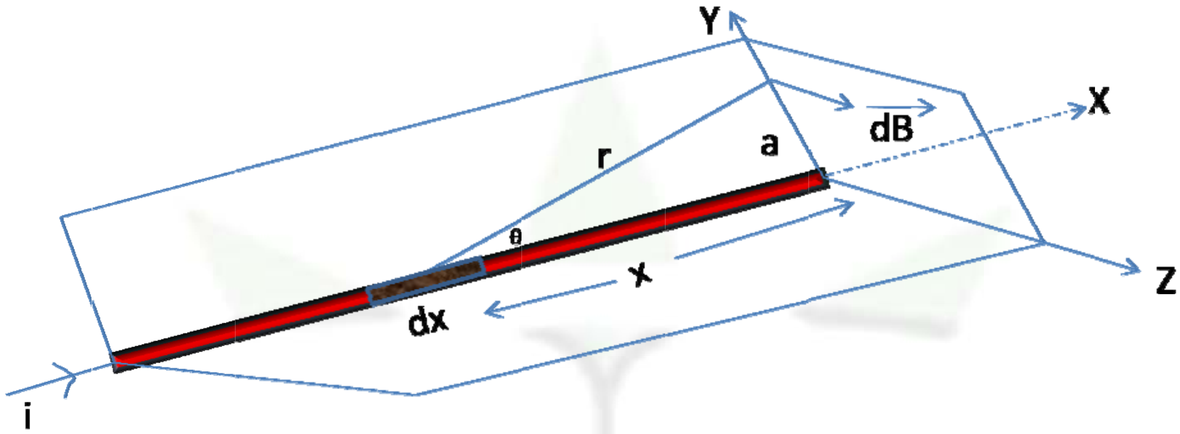
Biot – Savart Law: To find the magnetic field at a point due to an infinitely long straight conductor carrying current

Let us consider a long straight conductor carrying current. The point where the magnetic field is to be found is so close to the conductor that the conductor can be assumed to be infinitely long



Given i = current flowing through the conductor

a = the perpendicular distance of the given point from the conductor

Let us drop a perpendicular from the given point on the conductor, the foot of the perpendicular O is taken as origin. The length of the conductor in the direction of current is taken as X axis. The line OP is taken as Y axis, Z axis is chosen perpendicular to the X-Y plane as shown, so that X-Y-Z forms a right handed system of axes.

Let us consider an element of the conductor at A of length dx at a distance –x from origin O.

Let $AP = r, O\hat{A}P = \theta$

Applying Biots law the magnetic induction vector at P due to the current element at A

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i}{r^2}\vec{dx}\times\hat{r} \rightarrow (1)$$

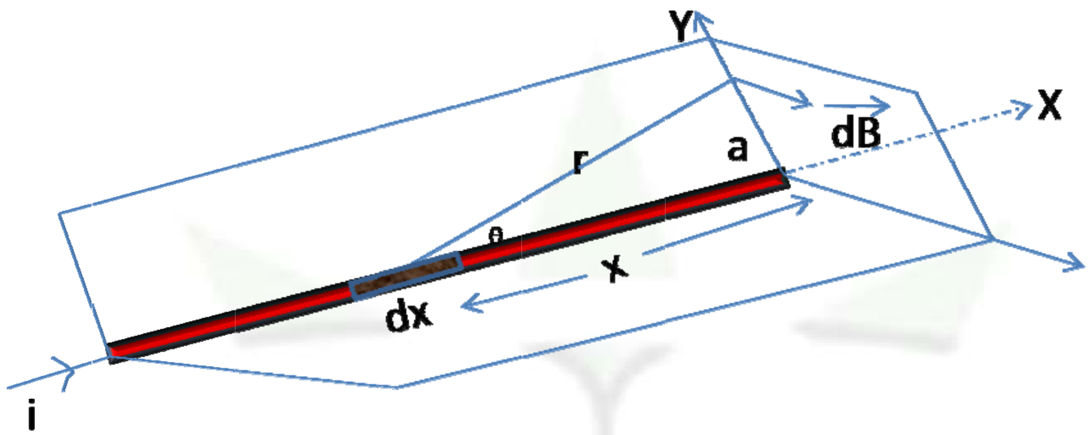
The magnitude of this induction vector

$$dB = \frac{\mu_0}{4\pi}\frac{i}{r^2}dx Sin\theta \rightarrow (2)$$

Applying the right handed curl rule for vector product the direction of this magnetic field is perpendicular to the X-Y plane along Z axis. If we consider the element at any point along the length of the conductor we find that the direction of the magnetic field is same for every position of the element i.e. Along Z axis in the figure. The vector addition reduces to scalar addition. Hence the resultant magnetic field at P due to the whole conductor can be obtained by integrating equation(2)

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{i}{r^2} dx Sin\theta \rightarrow (3)$$

Equation (3) contains three variables x, r & θ hence it is represented in terms of one variable only.



From $\Delta OAP, \text{cosec}\theta = \frac{r}{a}, r = a\text{cosec}\theta$

When $x = -\infty, \theta = 0$ and when $x = +\infty, \theta = 180^0 = \pi$

$$\therefore B = \frac{\mu_0}{4\pi} i \int_0^{\pi} \frac{a \cos ec^2\theta . \sin\theta}{a^2 \cos ec^2\theta} d\theta$$

$$B = \frac{\mu_0}{4\pi}\frac{i}{a}\int_0^{\pi} \sin\theta d\theta = \frac{\mu_0}{4\pi}\frac{i}{a}\left[-\cos\theta\right]_0^{\pi} = \frac{\mu_0}{4\pi}\frac{2i}{a} \rightarrow (5)$$

Equation (5) is known as Biot-Savart formula and gives the magnitude of the magnetic field at a point near a straight conductor carrying current.

The direction of the magnetic field which was found to be along Z axis in the figure can be stated as follows:

"Grasp the conductor in right hand with the thumb in the direction of current. The curl of the finger tips gives the direction of the magnetic field."