Q5. If a matrix has 5 elements, write all possible orders it can have.

Answer: Since a matrix of order $\mathrm{m} \times \mathrm{n}$ has mn elements, hence here mn=5 so possible values are are m=1, n=5 or m=5 and n=1. All possible ordered pairs (m, n) of positive integers whose product is 5 are 1×5 and 5×1.

Q6. Evaluate: $\int (ax+b)^3 \ dx$

Answer: Let $\mathrm{I} = \int (ax+b)^3 \ dx$

Let $u = ax + b$

Differentiation w.r.t. x, we get

$\frac{du}{dx} = a + 0 \Rightarrow \frac{du}{dx} = \frac{a}{1}$

$\therefore dx = \frac{du}{a}$

$\therefore \mathrm{I} = \int (u)^{\mathbf{3}} \ . \frac{du}{a}$

$= \frac{1}{a} \int (u)^3 \ dt$

$= \frac{1}{a}\left(\frac{u^4}{4}\right) + c$

$= \frac{1}{4a} \ u^4 + c \Rightarrow \frac{1}{4a} \ (ax+b)^4 + c$

Where c is constant of integration.

Q7. Evaluate: $\int \frac{dx}{\sqrt{1-x^2}}$

Answer: Let $\mathrm{I} = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$= [\sin^{-1} x]_0^1$

$= \sin^{-1}(1) - \sin^{-1}(0)$

$= \sin^{-1}\left(\sin\frac{\pi}{2}\right) - \sin^{-1}(0)$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

Q8. Write the direction – cosines of the line joining the points (1, 0, 0) and (0, 1, 1).

Answer:

We know that direction cosines of line joining points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$l = cos\alpha = \frac{x_2-x_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}},$$

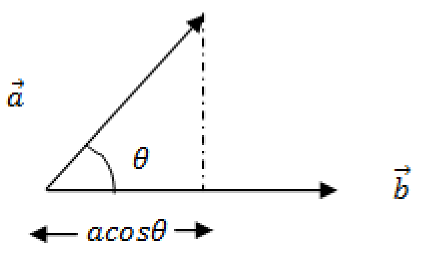
$$m = cos\beta = \frac{y_2-y_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}} \text{ and}$$

$$n = cos\gamma = \frac{z_2-z_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

Therefore direction cosines of the line joining the points (1, 0, 0) and (0, 1, 1) are $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Q9. Write the projection of the vector $\hat{\imath} - \hat{\jmath}$ on the vector $\hat{\imath} + \hat{\jmath}$.

Answer:



Projection of $\vec{a}$ on $\vec{b}$ is $acos\theta$

We know $\vec{a}.\vec{b} = abcos\theta$

Therefore $acos\theta = \frac{\vec{a}.\vec{b}}{b}$

Here $\vec{a} = \hat{\imath} - \hat{\jmath}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$

Now projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{0}{\sqrt{2}} = 0$

Q10. Write the vector equation of a line given by $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$.

Answer: The given line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$

We have standard equation of line as $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

On comparing we get $x_1 = 5, y_1 = -4,\ z_1 = 6$ and $a = 3,\ b = 7,\ c = 2$

Fixed point vector

$\vec{a} = x_1\,\hat{i} + y_1\,\hat{j} + z_1\,\hat{k}$

$= 5\hat{i} - 4\,\hat{j} + 6\,\hat{k}$

Direction vector

$\vec{b} = a\,\hat{\imath} + b\,\hat{\jmath} + c\,\hat{k}$

$= 3\,\hat{\imath} + 7\hat{\jmath} + 2\hat{k}$

$\therefore$ Vector equation of the given line is

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \left(5\hat{i} - 4\,\hat{j} + 6\,\hat{k}\right) + \lambda\left(3\,\hat{\imath} + 7\hat{\jmath} + 2\hat{k}\ \right)$