SelfStudy.in

SECTION - A

Q1. The binary operation *: $R \times R \to R$, is defined as a * b = 2a + b. Find (2*3)*4.

Answer: Given $a * b = 2a + b$

Therefore for a=2, b=3 we have $2 * 3 = 2 \times 2 + 3 = 7$

$\therefore (2 * 3) \times 4 = 7 * 4 = 2 \times 7 + 4 = 18$

Q2. Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

Answer: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \tan^{-1}\sqrt{3} - (\pi - \sec^{-1} 2)$

We know $\cos\frac{\pi}{3} = \frac{1}{2}$, so $\sec\frac{\pi}{3} = 2, \sec^{-1} 2 = \frac{\pi}{3}$ also $\tan\frac{\pi}{3} = \sqrt{3}$

$= \frac{\pi}{3} - \pi + \frac{\pi}{3}$

$= -\frac{\pi}{3}$

Q3. Find the value of x + y from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Answer: The given equation is

$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow 2x + 3 = 7; 2y - 4 = 14$

$\Rightarrow 2x = 4; 2y = 18$

$\Rightarrow x = 2; y = 9$

$\therefore x + y = 11.$

Q4. If If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$

Answer: Given : $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}_{3\times2}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}_{2\times3}$

$\Rightarrow B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}_{3\times2}$ (First column becomes first row and so on)

$\Rightarrow A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

Q5. Let A be a square matrix of order 3 X 3. Write the value of |2A|, where |A| = 4.

Answer: A is a 3 X 3 matrix (a square matrix has same number of rows and columns) as given 3X3 square matrix has 3 rows and 3 columns . |A| = 4.

$\Rightarrow |2A| = 2^3 . |A| = 8 \times 4 = 32$ [Taking 2 common from each column we get 2X2X2]

Q6. Evaluate $\int_0^2 \sqrt{4 - x^2}\ dx$

Answer: Let $I = \int_0^2 \sqrt{4 - x^2}\ dx = \int_0^2 \sqrt{2^2 - x^2}\ dx$

$We\ know\ integration\ by\ substitution\ can\ be\ used\ putting\ x = 2\sin\theta \Rightarrow dx = 2\cos\theta\ .d\theta$

$Also\ x = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

$x = 2 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}.$

$\therefore I = \int_0^{\pi/2} \sqrt{4 - 4\sin^2\theta}\ .2\cos\theta\ .d\theta$

$= 2\int_0^{\pi/2} 2\cos^2\theta\ d\theta = 2\int_0^{\pi/2} [1 + \cos 2\theta]\ d\theta$

$= 2\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2} = \pi.$