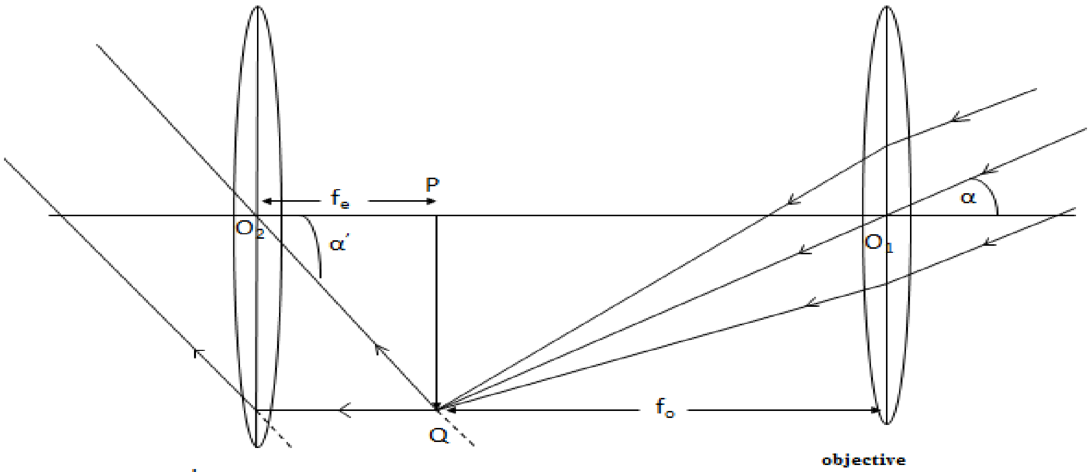
SelfStudy.in

Q. 22.

I. Draw a neat labelled ray diagram of an astronomical telescope in normal adjustment. Explain briefly it's working.

II. An astronomical telescope uses two lenses of powers 10 D and 1 D. What is its magnifying power in normal adjustment?

Answer: Ray diagram of an astronomical telescope (normal adjustment):



Working: Astronomical Telescope has two lenses an objective (focal length f_o) and eyepiece (focal length fe). The objective has a large focal length and much larger aperture than the eyepiece ($f_o>f_e$). When parallel rays from astronomical bodies at infinity incident on the objective after refraction at objective a real image is formed at the focal plane of objective at a distance f_o. For the eyepiece this image formed by objective serves as object, in normal adjustment the distance of the eyepiece is such that this PQ object is at its focal plane hence finally eyepiece forms the final inverted image at infinity as shown.

II. We know that P=1/f, the power of lens given = 10 D ∴ focal length $= \frac{100}{10} = 10$ cm.

The power of lens = 1 D, focal length $= \frac{100}{1} = 100$ cm.

Power in dioptre is converted to cm hence multiplied by 100.

We know that in astronomical telescope $f_o>f_e$. Therefore f_o=100 cm and fe=10 cm

Magnifying power $= \frac{f_o}{f_e} = \frac{100}{10} = 10$

Q. 23. In Young's double slit experiment, the two slits 0.15 mm apart are illuminated by monochromatic light of wavelength 450 nm. The screen is 0.1 m away from the slits.

a. Find the distance of the second (I) bright fringe, (II) dark fringe from the central maximum.
b. How will the fringe pattern change if the screen is moved away from the slits?

Answer: Given distance between two slits $(d) = 0.15\ mm = 0.15 \times 10^{-3}\ m = 15 \times 10^{-5}\ m$

$\lambda = 450\ nm = 450 \times 10^{-9}\ m = 4.5 \times 10^{-7}\ m,$

Distance of the screen from the light source $(D) = 1.0\ m$

a. (i) We know that for nth bright fringe

$x_n = \frac{n\lambda D}{d}$, distance of the second bright fringe(n=2)= $x_2 = \frac{2\lambda D}{d}$

$= \frac{2\times4.5\times10^{-7}\times1.0}{15\times10^{-5}} = \frac{2\times4.5}{15} \times 10^{-2} = 0.6 \times 10^{-2}\ \mathrm{m} = 6\ \mathrm{mm}$

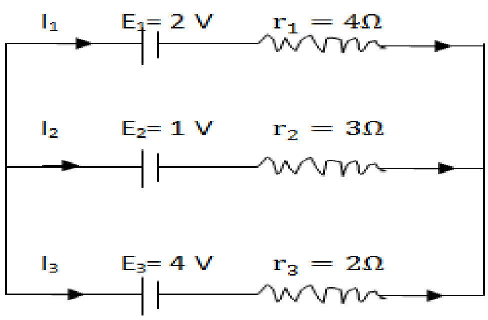
(ii) Distance of the second dark fringe = $x_n = \frac{(2n-1)\lambda D}{2d}$, $x_2 = \frac{3\lambda D}{2d}$

$= \frac{3\times4.5\times10^{-7}\times1.0}{2\times15\times10^{-5}} = \frac{3\times4.5}{30} \times 10^{-2} = 4.5 \times 10^{-2}\ \mathrm{m} = 4.5\ \mathrm{mm}$

b. We know that fringe width $(\beta) = \frac{\lambda D}{d}$

When screen is moved away, D increases. Therefore width of the fringes increases but the angular separation $\left(\frac{\lambda D}{d}\right)$ remains the same.

Q. 24. State Kirchhoff's rules. Use these rules to write the expressions for the currents I_1, I_2 and I_3 in the circuit diagram shown.



Answer: In complicated networks to find current flowing through different branches we apply Kirchhoff's law.

First law: The algebraic sum of currents meeting at a point is zero. i.e.$\Sigma i = 0$.

In the given circuit, Considering outward direction of current as positive and inward as negative then at the point F

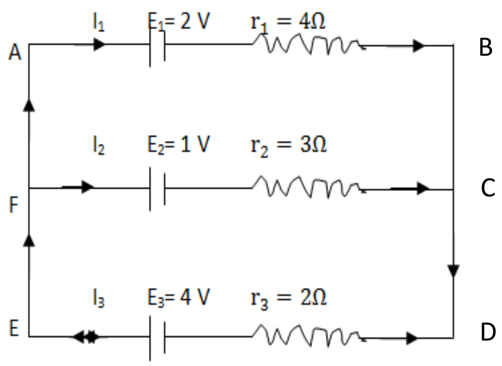
According to Kirchhoff's first law, at junction F

$I_1 + I_2 = I_3$

Second law: In a closed mesh of electrical conductors the algebraic sum of product of resistance and the respective current in the different branches is equal to the total e.m.f applied in the closed mesh.

$$\sum iR = \sum E$$

The given circuit diagram is



According to Kirchhoff's second law, in closed circuit ABCFA, we have

$-2 - 4\,I_1 + 3I_2 + 1 = 0 \quad [\because \sum IR + \sum E = 0]$

$-4\,I_1 + 3I_2 - 1 = 0 \quad \ldots\ldots\ldots\ldots(1)$

According to Kirchhoff's second law, in closed circuit FCDEF, we have

$-1 - 3\,I_2 - 2I_3 + 4 = 0$

$-3\,I_2 - 2I_3 + 3 = 0 \quad \ldots\ldots\ldots\ldots(2)$

According to Kirchhoff's first law, at junction F,

$I_1 + I_2 = I_3 \ldots\ldots\ldots\ldots(3)$

On solving eqns. (1), (2) and (3), we get

$I_1 = \frac{2}{13}, I_2 = \frac{7}{13}$ and $I_3 = \frac{9}{13}$

Q. 25. a. Write symbolically the β^-decay process of $^{32}_{15}P$.

b. Derive an expression for the average life of a radionuclide. Give its relationship with the half life.

Answer: (a) $^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + \beta^-$

We know that β^- is equivalent electron (-1 atomic number reduction but changes in mass number) $^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}\text{e}$

(b) **Average or Mean Life:** Average life is defined as the total life time of all the atoms of the element divided by the total number of atoms present in the sample of the element.

By definition, average life time of the radioactive element is $\tau = \frac{\int_0^{N_0} t\, dN}{N_0}$

Let N_0 = The number of radio active atoms present in the sample at an instant of time t = 0

N = The number of radio active atoms present in the sample at an instant of time t

N - dN = N_0 = The number of radio active atoms present in the sample at an instant of time t + dt

dN = The number of radio active atoms disintegrating in time dt at an instant of time t

$$\therefore -\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda dt \rightarrow (1)$$

λ = constant of proportionality known as decay constant or disintegration constant.

$$\frac{dN}{N} = -\lambda dt$$

Integrating both sides:

$$\log_e N = -\lambda t + A \rightarrow (2)$$

Where A is constant of integration. At t = 0, N = N_0 from equation (2)

$$\log_e N_0 = -\lambda \times 0 + A$$

$$or\ A = \log_e N_0 \rightarrow (3)$$

Putting equation (3) in (2)

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e \frac{N}{N_0} = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 e^{-\lambda t} \rightarrow (4)$$

Using equation (4) the number of radio active atoms present at any instant t can be calculated.

$N = N_0 e^{-\lambda t}$

$\text{From equation we find that at } t = 0, N = N_0 \text{ and } t = \infty, N = 0$

Half life period of a radioactive element is defined as the time during which half of the radioactive atoms present in the specimen disintegrated.

$\text{Let at } t = 0, N = N_0 \text{ be the number of radio active atoms present in the sample.}$

$\text{at } t = T, N = \frac{N_0}{2} = \text{Half of the original radio active atom present in the sample, then T is known as half life.}$

$\text{From equation (1)} : \frac{N_0}{2} = N_0 e^{-\lambda t}$

$\text{Taking log of both sides } -\log_e 2 = -\lambda T$

$T = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$

Equation gives the half life period.

When $N = 0$ then $t = \infty$ and when $N = N_0$ then $t = 0$

$$\therefore \tau = \frac{\int_{\infty}^{0} t.-\lambda N_0 e^{-\lambda t} dt}{N_0} = \lambda \int_{-\infty}^{0} t\, e^{-\lambda t} dt$$

$$\tau = \lambda \left[\frac{t\, e^{-\lambda t}}{\lambda} - \frac{e^{-\lambda t}}{\lambda^2}\right]_0^{\infty} \text{ (Using integration by parts)}$$

$$\tau = \lambda \left[(0-0) - \left(0 - \frac{1}{\lambda^2}\right)\right] = \lambda \times \frac{1}{\lambda^2} = \frac{1}{\lambda}$$

$$\text{Since } \lambda = \frac{0.693}{T}$$

$$\therefore \tau = \frac{1}{\frac{0.693}{t_{1/2}}} = \frac{t_{1/2}}{0.693} = 1.44 \text{ T}$$

Thus the average life period of a radioactive element (τ) is 1.44 times the half life period (T) of the element.