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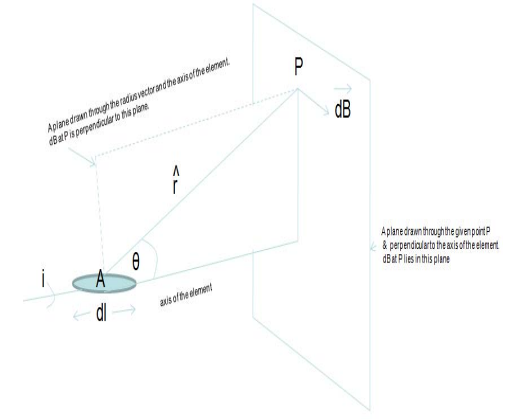
Q28. State Biot-Savart law, giving the mathematical expression for it.
Use this law to derive the expression for the magnetic field due to a straight conductor carrying current at a point along its axis.

How does a circular loop carrying current behave as a magnet?

Answer: Statement of Biot Savart Law: The magnitude of the magnetic field due to current element is directly proportional to the current, the element length and inversely proportional to the square of the distance of the field point. The direction of magnetic field is perpendicular to the plane containing current element and distance.



Given

i = current flowing through the element

dl = the length of the element of the conductor

r = AP = the distance of the given point from the element of the conductor

$\hat{r}$ = a unit vector along AP

$\hat{dl}$ = a vector of unit length along the axis of the element in the direction of current

θ = the angle between $\hat{r}$ and $\hat{dl}$

Let $\vec{db}$ = magnetic induction vector at the point P due to the current element

Experimentally it has been found that

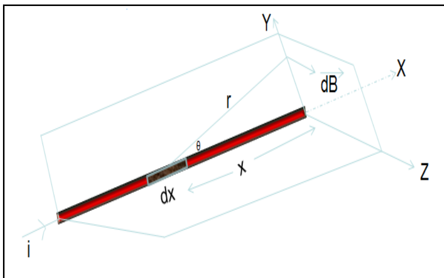
$$|\vec{db}| \propto i, |\vec{db}| \propto dl, |\vec{db}| \propto \frac{1}{r^2}, |\vec{db}| \propto \sin\theta$$

$$db \propto \frac{idl\sin\theta}{r^2}$$

$$db = \frac{\mu_0}{4\pi}\frac{i}{r^2}\left(\vec{dl} \times \hat{r}\right)$$

$$db = \frac{\mu_0}{4\pi}\frac{i}{r^3}\left(\vec{dl} \times \vec{r}\right)$$

To find the magnetic field at a point due to an infinitely long straight conductor carrying current.



Let AP = r, $O\hat{A}P = \theta$

Applying Biots law the magnetic induction vector at P due to the current element at A

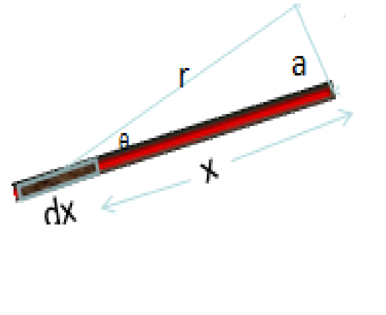
$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i}{r^2}\vec{dx} \times \hat{r} \rightarrow (1)$$

The magnitude of this induction vector

$$dB = \frac{\mu_0}{4\pi}\frac{i}{r^2}dxSin\theta \rightarrow (2)$$

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{i}{r^2}dx Sin\theta \rightarrow (3)$$

Equation (3) contains three variables x, r & θ hence it is represented in terms of one variable only.



$$\text{cosec}\theta = \frac{r}{a}, r = a\text{cosec}\theta$$

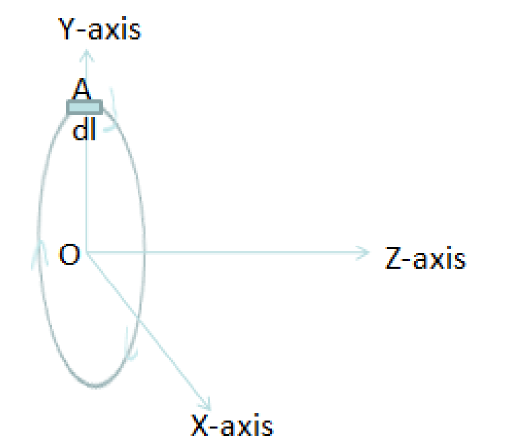
When $x = -\infty, \theta = 0$ and when $x = +\infty, \theta = 180^0 = \pi$

$$\therefore B = \frac{\mu_0}{4\pi} i \int_0^{\pi} \frac{a\cos ec^2\theta . \sin\theta}{a^2 \cos ec^2\theta} d\theta$$

$$B = \frac{\mu_0}{4\pi}\frac{i}{a}\int_0^{\pi} \sin\theta d\theta = \frac{\mu_0}{4\pi}\frac{i}{a}[-\cos\theta]_0^{\pi} = \frac{\mu_0}{4\pi}\frac{2i}{a} \rightarrow (5)$$

Equation(5) is known as Biot-Savart formula and gives the magnitude of the magnetic field at a point near a straight conductor carrying current. The direction of the magnetic field which was found to be along Z axis in the figure can be stated as follows:

"Grasp the conductor in right hand with the thumb in the direction of current. The curl of the finger tips gives the direction of the magnetic field."



The direction of magnetic field is along the axis of the coil which can be found as follows "Curl the finger of right hand in the direction of current in the coil the thumb gives the direction of the magnetic field".

Thus magnetic flux emitting for this magnetic field produced is along X axis then this side i.e coil facing +X axis will behave as North pole and negative X axis (X' axis) will behave as South pole. Hence coil (loop) behaves as a magnetic dipole.

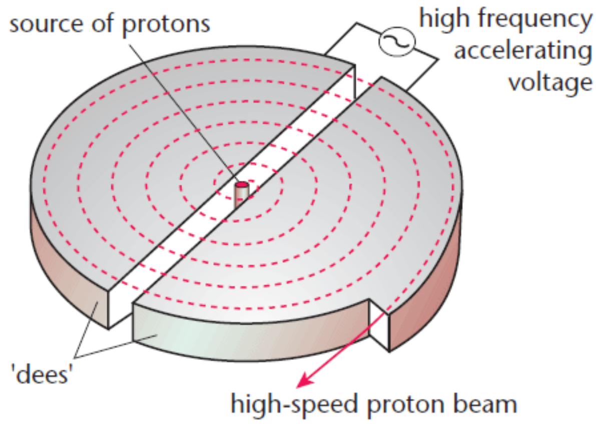
Or,

With the help of labelled diagram, state the underlying principle of a cyclotron. Explain clearly how it works to accelerate the charged particles.

Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle? Give reason.

Answer:

Principle: Cyclotrons accelerate charged particle beams using a high frequency alternating voltage which is applied between two "D"-shaped electrodes. An additional static magnetic field B is applied in perpendicular direction to the electrode plane. Thus charge particle acquires very high kinetic energy with the help of relative small electric field.



Working: A cyclotron consists of two D-shaped regions known as dees. In each dee there is a magnetic field perpendicular to the plane of the page. In the gap separating the dees there is a uniform electric field pointing from one dee to the other. When a charge is released from rest in the gap it is accelerated by the electric field and carried into one of the dees. The magnetic field in the Dee causes the charge to follow a half-circle that carries it back to the gap.

While the charge is in the dee the electric field in the gap is reversed, so the charge is once again accelerated across the gap. The cycle continues with the magnetic field in the dees continually bringing the charge back to the gap. Every time the charge crosses the gap it picks up speed. This causes the half-circles in the dees to increase in radius, and eventually the charge emerges from the cyclotron at high speed.

Theory: The centripetal force needed by the charged particle to move in circular track, is provided by the magnetic field.

m= mass of the charge particle, v=velocity, r=radius of the circular track, q=charge, B=Magnetic field

$\frac{mv^2}{r} = qvB$ or $v = \frac{qBr}{m}$, Now period of revolution= $T = \frac{2\pi r}{v} = \frac{2\pi r}{\frac{qBr}{m}} = \frac{2\pi m}{qB}$

Frequency $\nu = \frac{1}{T} = \frac{qB}{2\pi m}$, therefore the frequency of revolution is independent of energy of the particle.

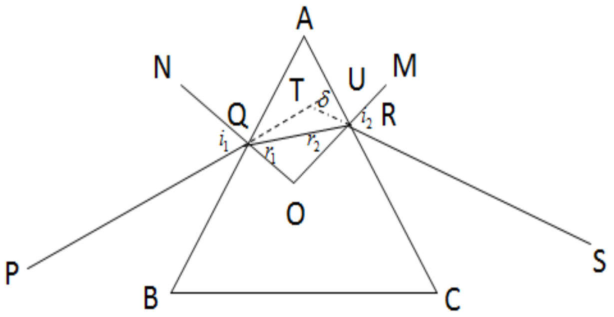
There is an upper limit on the energy acquired by the charged particle. The speed hence energy gain is maximum (at upper limit) when the radius of the moving path is equal to the radius of the dees.

Q29. (a) Draw a ray diagram to show refraction of a ray of monochromatic light passing through a glass prism.

Deduce the expression for the refractive index of a glass in terms of angle of prism and angle of minimum deviation.

(b) Explain briefly how the phenomenon of total internal reflection is used in fibre optics.

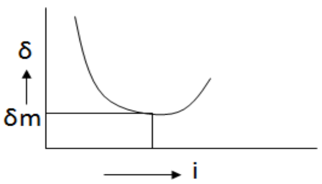
Answer (a): Refraction through prism



A = Angle of prism

Had there been no prism, the ray PQ would have gone straight along PQU, but due to the prism the ray finally goes along RS. Hence the prism has deviated the ray through the angle UTR=δ and is known as angle of deviation.

ON & OM are perpendiculars drawn on AB and AC respectively.



Let

$\angle PQN = i_1$ = angle of incidence

$\angle RQO = r_1$

$\angle SRM = i_2$ = angle of emergence

$\angle AQO = \pi/2 = \angle ARO$

In quodrilateral AQOR

Sum of two opposite angles $\angle AQO + \angle ARO = 180°$

Hence AQOR is a co cyclic quadrilateral

$\angle QAR + \angle QOR = 180°$

$A + \angle QOR = 180° \rightarrow (1)$

In ΔQOR

$r_1 + r_2 + \angle QOR = 180° \rightarrow (2)$

From equation (1) and equation (2):

$A = r_1 + r_2 \rightarrow (3)$

$\angle TQR = i_1 - r_1$

$\angle TRQ = i_2 - r_2$

$\delta = (i_1 - r_1) + (i_2 - r_2)$

$\delta = (i_1 + i_2) - (r_1 + r_2)$

$\delta = (i_1 + i_2) - A \rightarrow (4)$

From equation (4) we find that the angle of deviation δ depends on the angle of Incidence. Experimentally it is found that as the angle of incidence increases angle of deviation decreases, reaches a minimum value δm for a particular angle of incidence and then increases again.

The minimum value δm called the angle of minimum deviation. It can be analytically proved that when

$i_1 = i_2, r_1 = r_2$ i.e. the path of the ray through the prism is parallel to the base of the prism then only the angle of deviation is minimum.

Condiation for minimum deviation

$i_1 = i_2$ & $r_1 = r_2$

From equation (4)

$$\delta m = i + i - A$$

$$i = \frac{A + \delta m}{2}$$

$$A = r_1 + r_2 = r + r$$

$$A = 2r$$

$$r = \frac{A}{2}$$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A + \delta m}{2}\right)}{\sin\frac{A}{2}}$$

For a thin prism the angle of prism A is very small

$$\sin\frac{A}{2} \approx \frac{A}{2}$$

$$\sin\left(\frac{A + \delta m}{2}\right) \approx \frac{A + \delta m}{2} \approx \frac{A + \delta}{2}$$

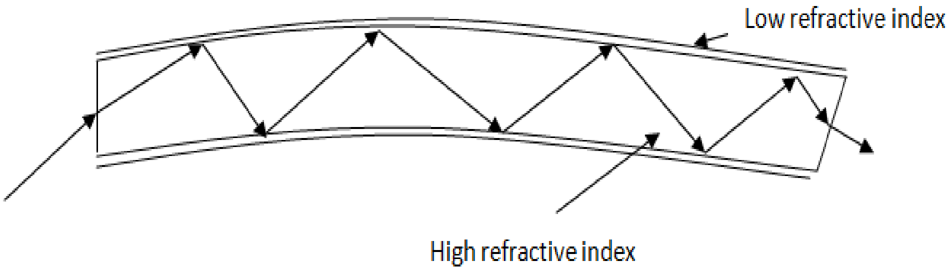
$$\therefore \mu = \frac{\frac{A + \delta}{2}}{\frac{A}{2}}$$

$$or\ A\mu = A + \delta$$

$$\text{or } \delta = A(\mu - 1) \rightarrow (6)$$

Equation (6) gives the deviation produced by thin prism.

(b) Total internal reflection is used in optical fibre: Optical fibre consists of a core and cladding. Refractive index of the material of the core is higher than that of cladding. When a signal, in the form of light, is directed into the optical fibre, at an angle greater than the respective critical angle, it undergoes repeated total internal reflections as shown along the length of the fibre.



Finally the signal comes out from the other end with almost negligible loss of intensity.

OR

(a) Obtain lens makers formula using the expression

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Here the ray of light propagating from a rarer medium of refractive index (n_1) to a denser medium of refractive index (n_2) is incident on the convex side of spherical refracting surface of radius of curvature R.

Answer:

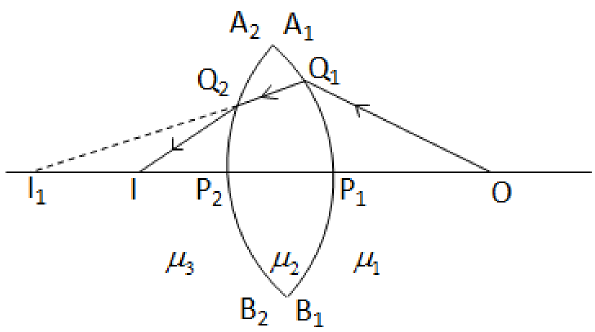


Fig-1

A_1B_1 & A_2B_2 are two spherically curved surfaces separating media of refractive indices μ_2, μ_1 and μ_3, μ_2 respectively. The medium bounded by the curved surface is denser than both the surrounding media i.e. ($\mu_1 < \mu_2 > \mu_3$). In figure-1 the concave faces bound the denser medium.

Ray diagram: A point object O is kept on the common principal axis in the medium of refractive index μ_1. A ray from O is incident at any point Q_1 on the curved surface A_1B_1 & afterrefraction since it is going from rarer to denser medium it bends towards the normal and is refracted along Q_1Q_2. Another ray from O incident along the the principal axis OP_1 being incident normally passes without any deviation along P_1P_2, these two refracted rays Q_1Q_2 and P_1P_2 meet at I_1 directly in figure-1. For the curved surface A_2B_2, Q_1Q_2 & P_1P_2 are incident rays since they appear to converge (in figure-1) hence I_1 serves as object for A_2B_2. Since Q_1Q_2 goes from denser to rarer medium bends away from the normal where as P_1P_2 passes without any deviation & meet at I. Hence I is the image of the object O formed by the double spherical surface.

Calculation: We know that for refraction at a single spherical surface with object in the rarer medium

$$\frac{\mu_o}{u} + \frac{\mu_i}{v} = \frac{\mu_i - \mu_o}{r} \rightarrow (1)$$

For refraction at curved surface A_1B_1
Refractive index of the object medium $\mu_o = \mu_1$
Refractive Index of the image medium $\mu_i = \mu_2$
Object distance $P_1O=u$
Image distance$=P_1I=v_1$
Radius of curvature $=r_1$
Using equation (1)

$$\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{r_1} \rightarrow (2)$$

For refraction at curved surface A_2B_2
Refractive index of the object medium $\mu_o=\mu_2$
Refractive index of the image medium $\mu_i=\mu_3$
Since $\mu_2 > \mu_3$ therefore $\mu_o > \mu_i$, since the object is in denser medium we have to
Consider sign correction before using the equation (1).
Figure- 1 : Object distance = $P_2I_1=u'$(say)=Negative
Image distance = $P_2I=v$=Positive
Radius of curvature $=r_2$= Negative

Using equation (1)

$$\frac{\mu_2}{-u'} + \frac{\mu_3}{v} = \frac{\mu_3 - \mu_2}{-r_2}$$

$$\frac{\mu_2}{-u'} + \frac{\mu_3}{v} = \frac{\mu_2 - \mu_3}{r_2} \rightarrow (3)$$

adding equation (2) & (3)

$$\frac{\mu_1}{u} + \frac{\mu_2}{v_1} - \frac{\mu_2}{u'} + \frac{\mu_3}{v} = \frac{\mu_2 - \mu_1}{r_1} + \frac{\mu_2 - \mu_3}{r_2}$$

$$v_1 = P_1 I = P_2 I \pm P_1 P_2 = u' \pm t$$

$P_1 P_2 = t =$ thickness of the lens for thin lens $v_1 = u'$

$$\frac{\mu_1}{u} + \frac{\mu_3}{v} = \frac{\mu_2 - \mu_1}{r_1} + \frac{\mu_2 - \mu_3}{r_2}$$

Above equation gives the relation between the object distance image distance and radius of curvature for refraction through a lens, when the two surrounding media are different.

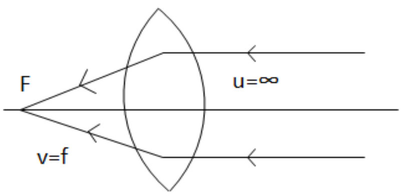
Special Case I: Let both the media surrounding the lens be same $\mu_1 = \mu_3$

$$\frac{\mu_1}{u} + \frac{\mu_1}{v} = \frac{\mu_2 - \mu_1}{r_1} + \frac{\mu_2 - \mu_1}{r_2}$$

$$\frac{1}{u} + \frac{1}{v} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{u} + \frac{1}{v} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

If rays are incident parallel to the principal axis (u=∞), then the point on which image is formed is said to be the FOCUS. The distance of the focus from the optical center is known as focal length (f).



From equation

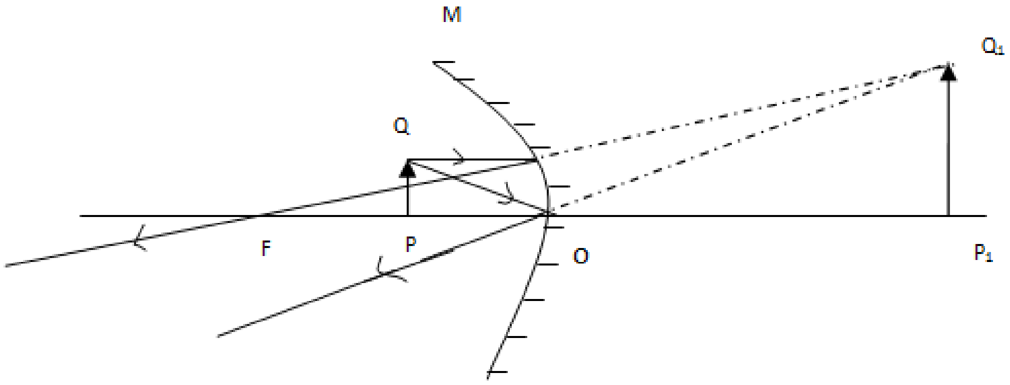
$$\frac{1}{\infty} + \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Equation gives the focal length of lens when the surrounding medium is other than air. Replacing μ2 by n_2 and μ_1 by n_1 ,r_1 by R_1 and R_2 by r_2 considering direction of curved surface opposite we get

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(b) Draw any ray diagram to show the image formation by a concave mirror when the object is kept between its focus and the pole. Using this diagram, derive the magnification formula for the image formed.



PQ is the object kept between the focus F and the pole o of the concave mirror, P_1Q_1 image formed by the mirror as shown.

Magnification Produced:

ΔPOQ and ΔP_1OQ_1 are $similar$.

$$\frac{P_1Q_1}{PQ} = \frac{OP_1}{OP}$$

OP1= image distance = -v (as per modern sign convention image distance measured against refracted rays taken as negative)

OP = object distance =+ u (as per modern sign convention object distance measured against incident rays taken as positive)

So magnification (m):

$$m = \frac{OP_1}{OP} = -\frac{v}{u}$$

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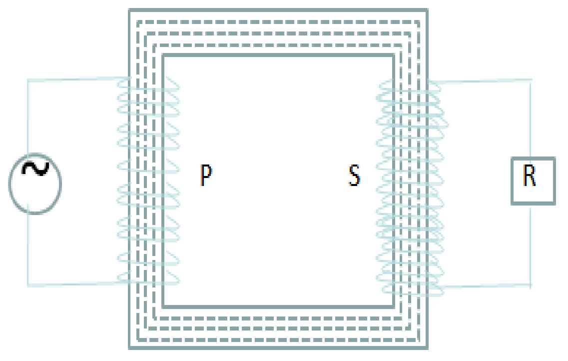
Q30. (i) With the help of a labelled diagram, describe briefly the underlying principle and working of a step up transformer.

Answer:



Principle: It is a device which converts high voltage, A.C. into low voltage A.C. and vice versa. It is based upon the principle of *mutual induction*. When alternating current passed through a coil (Primary), an induced e.m.f. is set up in the neighbouring coil(Secondary).

Construction: A transformer consists of two coils of many turns of insulated copper wire wound on a closed laminated iron core. One of the coils known as primary (P) is connected to A.C. supply. The other coil known as Secondary (S) is connected to the load.

Working: It consists of two coils known as Primary (P) and Secondary(S) having different no. of turns and are would on two opposite arms of a thick laminated iron core. The alternating emf to be transformed is connected across the primary coil. The varying current flowing through the primary coil produces varying magnetic lines of force. The magnetic ion core provides an easy path for the flow of lines of force hence the lines of force flowing through the core cuts the secondary coil and induces an emf across the secondary. An induced current flow through the secondary coil and thus a potential drop is obtained across the resistance which is known as output voltage.

Theory: Given

N_p and N_s are number of turns in the primary and secondary respectively

V_p and V_s are their respective voltages.

$\therefore V_p = -N_p \frac{d\phi}{dt}$ and $V_s = -N_s \frac{d\phi}{dt}$

Where, N_p and N_s are number of turns in the primary and secondary respectively and V_p and V_s are their respective voltages.

$\therefore \frac{V_s}{V_p} = \frac{N_s}{N_p}$

This ratio $\frac{N_s}{N_p}$ is called the turns ratio.

In a step − up transformer : $N_s > N_p$, *so* $V_s > V_p$.

(ii) Write any two sources of energy loss in a transformer.

Answer: Sources of energy loss in transformer

I. **Eddy currents:** The alternating magnetic flux induces eddy current in the iron core and causes heating. This effect is *reduced* by having a laminated core.

II. **Hysteresis:** This is due to repeated magnetisation and demagnetisation of the iron core. It can be reduced by using a magnetic material having low hysteresis loss.

(iii) A step up transformer converts a low input voltage into a high output voltage.
Does it violate law of conservation of energy? Explain.

Answer: A transformer does not violate law conservation of energy. In step up transformer voltage increased but at the same time current reduced, in step down transformer voltage decreased but current increased, thus the product of current and voltage that is power remains same in input and output provided there is no loss.

OR

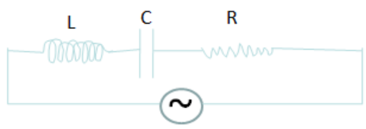
Derive an expression for the impedance of a series LCR circuit connected to an AC supply of variable frequency.

Plot a graph showing variation of current with the frequency of the applied voltage.

Explain briefly how the phenomenon of resonance in the circuit can be used in the running mechanism of a radio or a TV set.

Answer:

L-C-R circuit :



We know that in capacitor (C) current leads emf by π/2 , in inductance current lags emf by π/2 (L) and in resistance (R) current is in phase with emf.

If V_L, V_c, V_R and V represents the voltage across the inductor, capacitor, resistance and source.

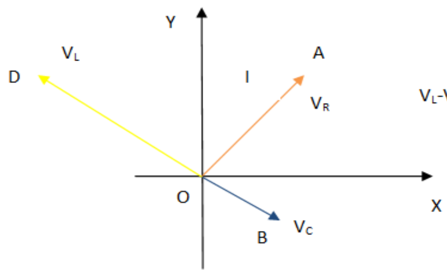


Fig-1

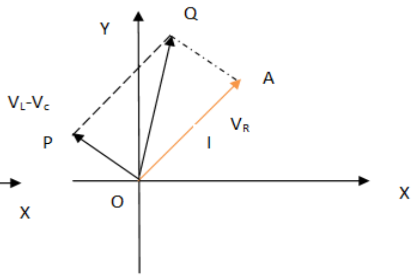


Fig-2

For the above vector diagram, Let the direction of current be along $\overrightarrow{OA}$, therefore

V_R= IR can be represented along $\overrightarrow{OA}$.

V_L=IX_L=IωL , since it lags current by π/2 , represented along $\overrightarrow{OD}$.

V_c=IXc=I/ωC, since current leads by π/2, represented along$\overrightarrow{OB}$.

Therefore V_c and V_L are 180^0 apart, resultant obtained by subtraction= V_L- V_C=$\overrightarrow{OP}$ (shown in fig-2)

Now from ΔOQP: $OQ^2 = OP^2 + PQ^2$, $V^2= V_R^2+(V_L -V_c)^2= I^2[R^2 +(\omega L- 1/\omega C)^2]$ Or $\frac{V}{I} = Impedance(Z)$

The impedance of the circuit

$$z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

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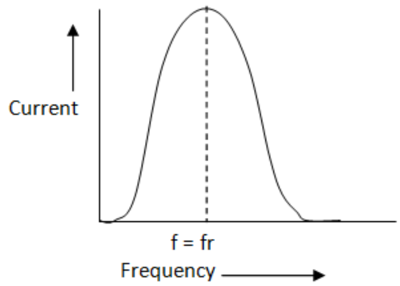
The given graph showing variation of current with the frequency of the applied voltage.



The radio and TV receiver sets are the practical applications of series resonant circuits. Signals of several different frequencies are available in air. The capacitance of the capacitor in tuning circuit is varied by turning the tuning knob of the radio/TV set. Thus the frequency of the LCR circuit varied till it matches the frequency of the derived signal.

A series resonant circuit allows maximum current through it. So, it is called acceptor circuit. A series resonant circuit is that circuit in which inductance L, capacitance C and resistance R are connected in series. The impedance of this circuit has a minimum value and the current through the circuit is maximum.

Note: Most of answers are written in descriptive way to make you comfortable for self study, however while writing in exam you need to write in brief with proper understanding to complete your answer sheet in time.