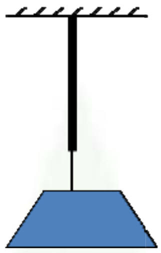
Elasticity: A wire is clamped at one end and loaded at its free end. It is found that the length of the wire changes. The force is known as deforming force, the changes known as deformation. If after removal of the deforming force the body regains its original length the body is said to be perfectly elastic.



If after the removal of the deforming force the body retains its changed form completely the body is said to be perfectly plastic.

Hooke's law: For small deformation the deformation produced is proportional to the deforming force.

Stress: The total external force applied per unit area is known as stress.

Stress = F/A Unit: Newton /m^2, Dimension: [M L^{-1} T^{-2}]

Since pressure is also defined as the force per unit area hence stress & pressure have same unit & dimension. Stress can be defined into three

(1) Normal Stress
(2) Tensile Stress
(3) Shearing or Tangential Stress

Normal Stress = Force / Area of the surface to which the force is applied

Tensile (longitudinal) Stress: When equal forces are applied simultaneously along all the free directions of a body resulting a volume change the force per unit surface area is known as tensile stress.

Shearing Stress or Tangential stress: Force per unit area of the surface to which the force is parallel producing a change in shape.

Shearing Stress = Force / area of the surface to which the force is parallel

Strain: The ratio of the change in dimension (such as length, volume ...) to the original dimension is known as strain.

$$Strain = \frac{l}{L}$$

$$Strain = \frac{v}{V}$$

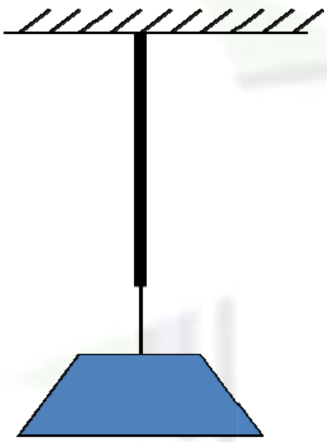
Since it is the ratio of two same physical quantities it has no unit and dimension. Depending upon the shape of the body and the nature of the force applied we have three different types of strain.

(1) Linear Strain

(2) Volume Strain

(3) Shearing Strain

Linear Strain: The ratio of change in length to the original length



(i)Longitudinal Strain

(ii) Lateral Strain

L & D = Original length and diameters

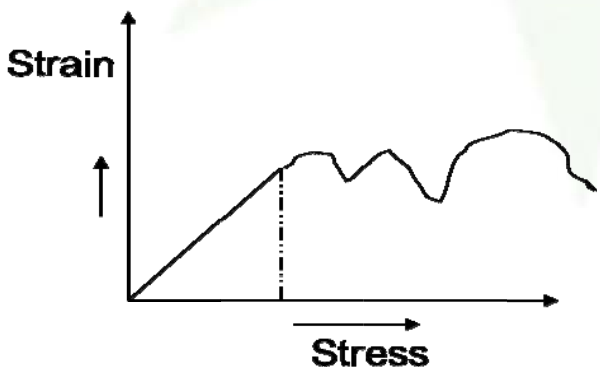
l & d = Change in length and diameter

l / L = Linear strain produced along the direction of applied force and is known as longitudinal strain.

d / D = The linear strain produced along the direction perpendicular to the force applied is known as lateral strain.

Hooke's Law: Within the elastic limit the strain produced is proportional to the stress applied and the ratio of stress to strain is constant known as modulus of elasticity or coefficient of elasticity.

It is found that up to a certain value of stress; strain varies linearly with stress, above that particular value of stress the linear relationship ceases to hold good and is known as elastic limit.



Since strain has no unit and dimension hence modulus of elasticity has same unit and dimension as stress Newton / m^2 , [M L-1 T-2]

Young's Modulus (Y): Let us consider a wire clamped at one end and loaded at its free end. The length of the wire increases (the diameter or breadth of the wire decreases).

Given: L = Original length of the wire

A = Area of cross section of the wire

F = the force applied

l = the change in length of the wire in the direction of force applied

F/A = Normal Stress, l / L = Longitudinal strain.

Applying Hook's law, within the elastic limit the longitudinal strain is proportional to the normal stress & the ratio of normal stress to the longitudinal strain is a constant known as Young's modulus of elasticity (Y).

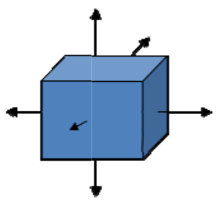
$$\frac{\text{Normal Stress}}{\text{Longitudinal Strain}} = \frac{F/A}{l/L} = Y$$

$$Y = \frac{F/A}{l/L} = \frac{F.L}{A.l}$$

If L = 1 m, A = 1 m^2, l = 1 m then Y = F

Hence Young's modulus of a material can be defined as a force required to stretch a wire of that material having unit length (L = 1m) & unit area of cross section (A=1 m^2) by unity (l = 1m)

(2) **Bulk Modulus** (k): Let us consider a cube ABCDEFGH having a volume V. Let Equal force per unit area P (Stress) is applied simultaneously at the center of all the sides faces along perpendicular to the surface as shown. Then P is known as tensile stress. The volume of the cube will increase.



Let v = change in volume of the cube

Volume strain $= \frac{v}{V}$

Applying Hook's law within the elastic limit the volume strain is proportional to the tensile stress and the ratio of tensile stress to volume strain is constant known as bulk modulus of elasticity.

Bulk Modulus $k = \frac{p}{v/V} = \frac{pV}{v}$

Physical significance:

Given $K_{Iron} > K_{Copper}$

$$k_{Iron} > k_{Copper}$$
$$\frac{P_1 V}{v} > \frac{P_2 V}{v}$$
$$P_1 > P_2$$

Substance having greater bulk modulus of elasticity is difficult to be compressed, among solid liquid and gas, liquid is most difficult to be compressed and hence liquid possesses high value of k, where as gas can be easily compressed and possesses low value of k

Gas possesses two values of bulk modulus of elasticity.

The volume of a gas can be changed by changing pressure but for the same change in pressure the change in volume is different under condition

(i) Isothermal Condition (T = constant)

(ii) Adiabatic condition ($\Delta Q = 0$)

Hence gas possesses two value of k

(i) Isothermal bulk modulus of elasticity (E_i)

(ii) Adiabatic bulk modulus of elasticity (E_a)

Let P = Pressure of the gas

V = Volume of the gas at pressure P.

Let the pressure be changed to (P+ dp) the volume decreases to V – dv

Tensile stress = dp

Volume Strain $= -\frac{dv}{V}$

$$k = \frac{dp}{-\frac{dv}{V}} = -\frac{dp}{dv}V$$

$$E = -\frac{dp}{dv}V \longrightarrow (1)$$

(i) Isothermal bulk modulus of elasticity E_i

$$E_i = -\frac{dp}{dv}V \longrightarrow (2)$$

We know that under isothermal condition pressure & volume of a gas are related by PV = Constant

Differentiating $$Pdv + dpV = 0$$

$$-\frac{dp}{dv}V = P \longrightarrow (3)$$

Putting equation (3) in (2) $E_i = P \longrightarrow (4)$

(ii) Adiabatic bulk modulus of elasticity (E_a):

We know that under adiabatic condition pressure & volume of a gas are related by

$$PV^{\gamma} = \text{Constant}$$

$$\gamma = \frac{C_p}{C_v} = \text{Ratio of two specific heats of gas(constant)}$$

Differentiating:

$$P\gamma V^{\gamma-1}dv + dpV^{\gamma} = 0$$

$$V^{\gamma-1}\left[\gamma Pdv + dpV\right] = 0$$

$$\gamma Pdv = -dpV$$

$$\gamma P = -\frac{dp}{dv}V$$

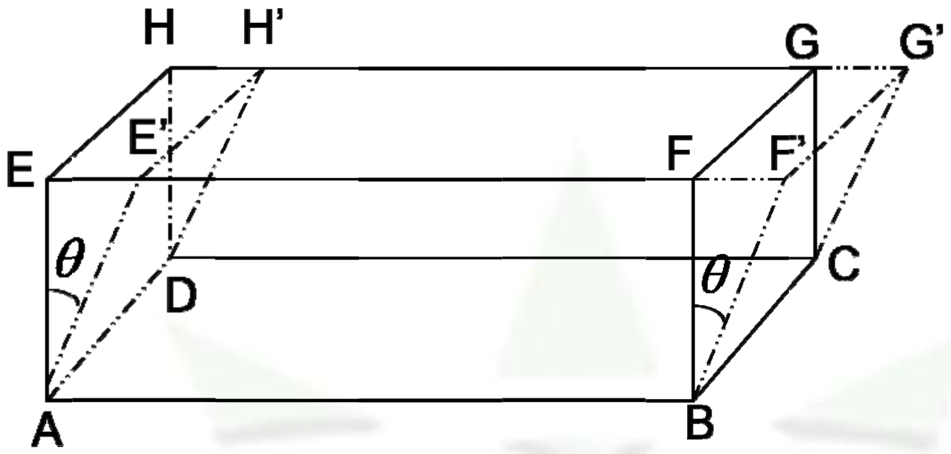
$$E_a = \gamma P \longrightarrow (5)$$

$$\frac{E_a}{E_i} = \frac{\gamma P}{P} = \gamma$$

$$\because \gamma > 1$$

$$\therefore E_a > E_i$$

Rigidity Modulus (n): Let us consider a rectangular block ABCDEFGH, who's lower surface ABCD is rigidly clamped. A force f is applied parallel to the surface EFGH which is parallel to the clamped surface. The shape changes to ABCEDE'F'G'H'. The size i.e. the length, breadth, volume etc does not change. The body is said to be sheared.



The applied force f per unit area of the surface to which the force is parallel is known as shearing stress or tangential stress = F/a

Where a = area of the surface EFGH

$$\angle EAE' = \angle HDH' = \angle FBF' = \angle GCG' = \theta = \text{Shearing strain or angle of shear}$$

Applying Hook's law within elastic limit ratio of shearing stress to shearing strain is constant known as rigidity modulus.

$$\text{Rigidity Modulus } (\eta) = \frac{f/a}{\theta}$$

$$\text{Let EE}' = l, AE = L$$

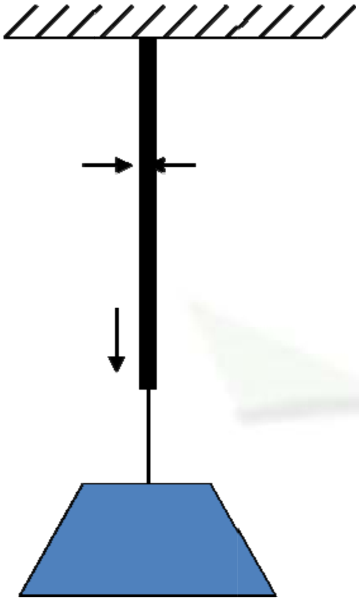
l = relative displacement between two layers which are separated by a distance L

$$\tan\theta = \frac{l}{L}, \text{since } \theta \text{ is small}, \tan\theta \approx \theta$$

$$\text{shearing strain } \theta = \frac{l}{L}$$

$$\therefore \eta = \frac{f/a}{l/L}$$

Poisson's Ratio (σ): A wire is clamped at one end and loaded at its free end. The length of the wire increases and the breadth or the diameter of the wire decreases. Thus every longitudinal strain is accompanied by a lateral strain (unless prevented).



L & D = original length and diameter of the wire.

l & d = change in length and diameter of the wire.

$$\therefore \text{Longitudinal strain} = l/L$$

$$\text{Lateral strain} = d/D$$

Within elastic limit the ratio of lateral strain to longitudinal strain is constant and is known as Poisson's ratio.

Poisson's Ratio (σ) $= -\dfrac{\text{lateral strain}}{\text{longitudinal strain}} = -\dfrac{d/D}{l/L}$

The negative sign indicates that the two strains are just opposite in nature i.e. when one is extensional other is contraction.