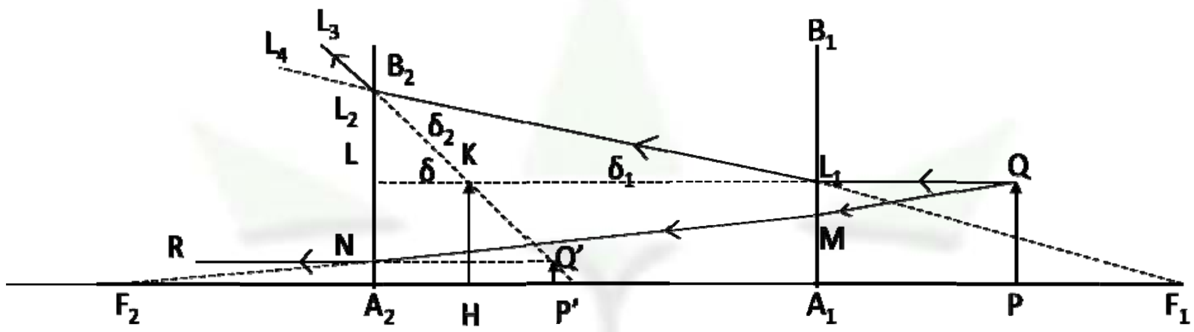
Equivalent focal length of two lenses kept separated by a small distance:

We now derive the relation taking concave lenses. Let us consider two concave lenses whose sections A_1B_1 and A_2B_2 are shown in the figure. The lenses are placed coaxially by a small distance.



An extended object PQ is kept in front of the lens L_1 on the common principal axis.

Ray diagram: From Q a ray QL_1 is incident on the lens A_1B_1, parallel to the principal axis and is refracted along L_1L_2, such that it appears to diverge from F_1 the focus of the lens A_1B_1. The ray L_1L_2 is further diverged by lens A_2B_2 and goes along L_2L_3 consider another ray QM which is incident on A_1B_1 in such a way that after refraction along MN it goes towards the focus F_2 of the lens A_2B_2 and hence after refraction by lens A_2B_2 it goes along NR parallel to the principal axis. These two final refracted rays L_2L_3 & NR are produced back to meet at Q'. Hence P'Q' is the image of the object PQ formed by the combination of lenses.

If a single lens can produce an image P'Q' of the object PQ then it is said to be the equivalent lens. From the ray diagram we see if corresponding to the incident ray QL_1,L_2L_3 becomes the refracted ray then only P'Q' will be the image of the object PQ. So we must search out a lens which will satisfy this. From the diagram we see that if a lens be placed at HK where K is the intersection of incident ray QL_1 and refracted ray L_2L_3 and if F be the focus of that lens then only L_2L_3 becomes the refracted ray corresponding to the incident ray QL.

Thus HK is the position of the equivalent lens and HF is the focal length of the equivalent lens.

Calculation: Given f_1 & f_2 = Focal length of lenses A_1B_1 and A_2B_2 respectively.

a = the distance of separation between the two lenses.

$PQ=y_1=A_1L_1=HK=A_2L$ and $A_2L_2= y_2$

Let $\delta_1 = \angle LL_1L_2$ = the deviation of the ray parallel to the principal axis produced by lens $A_1B_1 = \angle A_1F_1L_1$

From $\Delta A_1F_1L_1$: $\tan\delta_1 = \dfrac{A_1L_1}{A_1F_1} = \dfrac{y_1}{-f_1}$

since deviation is small $\tan\delta_1 \approx \delta_1 = -\dfrac{y_1}{f_1} \rightarrow (1)$

The negative sign simply indicates that the deviation is away from the principal axis.

$\angle L_4L_2L_3 = \delta_2$ = the deviation of the ray L_1L_2 produced by the lens A_2B_2

since δ_1 is very small ray L_1L_2 is nearly parallel to the principal axis and hence

following equation (1) we can similarly have $\delta_2 = -\dfrac{y_2}{f_2} \rightarrow (2)$

Let HF = f = the focal length of the equivalent lens.

Let $\delta = \angle LKL_3$ = the deviation of the ray QL incident parallel to the pricipal axis produced by equivalent lens HK = $\angle HFK$

$\therefore \Delta HFK$; $\tan\delta \approx \delta = \dfrac{HF}{HK} = -\dfrac{y_1}{f} \rightarrow (3)$

From ΔL_1KL_2; $\angle LKL_2 = \angle KL_1L_2 + \angle KL_2L_1$

$\delta = \delta_1 + \delta_2 \rightarrow (4)$

Putting equation (1), (2) & (3) in (4)

$$-\frac{y_1}{f} = -\frac{y_1}{f_1} + \frac{-y_2}{f_2} \quad or \quad \frac{y_1}{f} = \frac{y_1}{f_1} + \frac{y_2}{f_2} \rightarrow (5)$$

To find y_2 :

$y_2 = A_2L_2 + A_2L + LL_2 = y_1 + LL_2$

From ΔLL_1L_2 $\tan\delta_1 \approx \delta_1 = \dfrac{LL_2}{LL_1}$

or $LL_2 = \dfrac{ay_1}{f_1}$

$$\therefore y_2 = y_1 + y_1\frac{a}{f_1} \quad \text{or} \quad y_2 = y_1\left[1 + \frac{a}{f_1}\right] \rightarrow (6)$$

Putting equation (6) in (5)

$$\frac{y_1}{f} = \frac{y_1}{f_1} + \frac{y_1}{f_2} + \frac{ay_1}{f_1 f_2} \quad \text{or} \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2} \rightarrow (7)$$

Equation (7) gives the equivalent focal length of the two lenses kept separated by a small distance.

Position of the equivalent lens:

Let $A_2H = x =$ the distance of the equivalent lens from the lens A_2B_2

From ΔLKL_2; $\tan\delta \approx \delta = \frac{LL_2}{LK} = \frac{LL_2}{x}$

$$or\ x = \frac{LL_2}{\delta} = \frac{ay_1/f_1}{y_1/f} = \frac{af}{f_1} \rightarrow (8)$$

Equation(8) gives the position of the equivalent lens.

Discussion (1): If the two lenses are kept in contact a = 0 then using equation (7) equivalent focal length

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \rightarrow (9)$$

(2) Power of the equivalent lens: $P = P_1 + P_2 + aP_1P_2$