Gravitational field of earth:

Let us consider a particle on or outside the surface of earth.

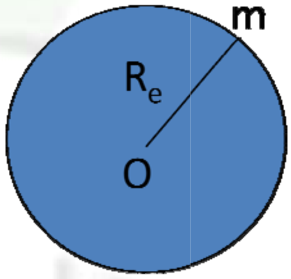
Given M_e and R_e = Mass & Radius of earth respectively

m = mass of the given particle on or outside the surface of earth

Since the given particle lies on or outside the surface of earth to find the force on the particle we can assume the mass of earth (solid sphere) to be concentrated at its center.

Gravitational force with which earth attracts the particle towards its center according to the Newton's law of gravitation

$$F = \frac{GM_e m}{R_e^2}$$



F = Weight of the particle. This force towards the center of earth produces acceleration towards the center of earth and since this acceleration is produced by the gravitational force it is known as acceleration due to gravity & is represented by g

$$W = F = mg = \frac{GM_e m}{R_e^2}$$

$$g = \frac{GM_e}{R_e^2}$$

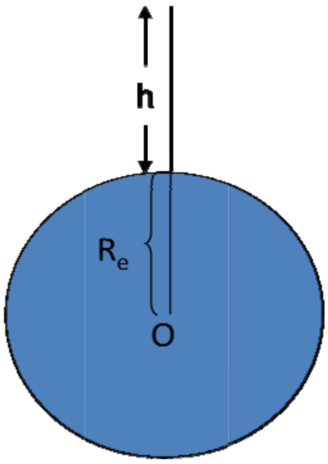
Although from equation (3) g appears to be a constant but we now discuss the factors with which g changes the acceleration due to gravity g varies with following.

(1) Variation of g with height (altitude)

(2) Variation of g with depth

(3) Variation of g with latitude

Variation of g with altitude:



Given Me and Re = Mass and Radius of earth

$$g = \frac{GM_e}{R_e^2}$$

Acceleration due to gravity on the surface of earth. Let us consider a particle of mass m at P at a height h above the surface of earth.

Let g' be the acceleration due to gravity at P. Since the given particle lies outside the surface of earth (solid sphere) assuming the mass of earth to be concentrated at its center, we get the force with which earth attracts the particle towards its center.

$$W = F = \frac{GM_e m}{(R_e + h)^2} = mg'$$

$$g' = \frac{GM_e}{(R_e + h)^2} = \frac{GM_e}{R_e^2\left(1 + {h}/{R_e^2}\right)^2} = \frac{GM_e}{R_e^2}\left(1 + {h}/{R_e^2}\right)^{-2}$$

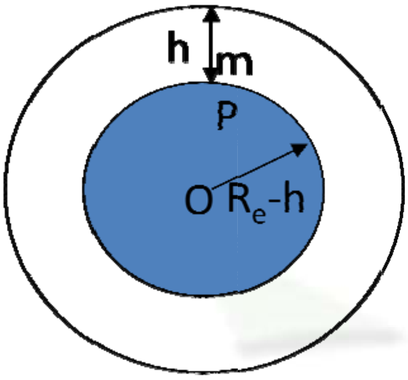
$$g' = \frac{GM_e}{R_e^2}\left[1 + (-2)\frac{h}{R_e} + ...neglected\right]$$

$$g' = g\left[1 - \frac{2h}{R_e}\right]$$

Thus we find that acceleration due to gravity decreases as we move above the surface of earth.

(2) **Variation of g with depth**

Given M_e & R_e = Mass & Radius of earth



$$\rho_e = \frac{3M_e}{4\pi R_e^3}$$

$$g = \frac{GM_e}{R_e^2} = \frac{G\frac{4}{3}\pi R_e^3 \rho_e}{R_e^2}$$

$$g = G\frac{4}{3}\pi R_e \rho_e$$

Acceleration due to gravity on the surface of earth.

m = mass of the given particle at a depth h below the surface of earth.

Let g' = Acceleration due to gravity at P

Since the given point lies inside the earth (solid sphere) we cannot assume the mass of earth to be concentrated at its center.

We can assume the earth to be made up of two parts

(1) The inner solid sphere of center O , Radius (R_e- h) passing through the given point P
(2) The concentric thin shell (h is small)

Since the intensity at any internal point of a thin shell is zero hence the force on the particle of mass m due to the outer thin shell is zero. Hence only the inner solid sphere is effective in exerting force on the particle.

Mass of the inner solid sphere $= \frac{4}{3}\pi (R_e - h)^3 \rho_e$

Since the given point P lies on the surface of the inner solid sphere we can assume the mass of it to be concentrated at its center.

Force on the particle $F = \frac{G\frac{4}{3}\pi (R_e - h)^3 \rho_e \times m}{(R_e - h)^2} = mg'$

$$g' = G\frac{4}{3}\pi (R_e - h)\rho_e$$

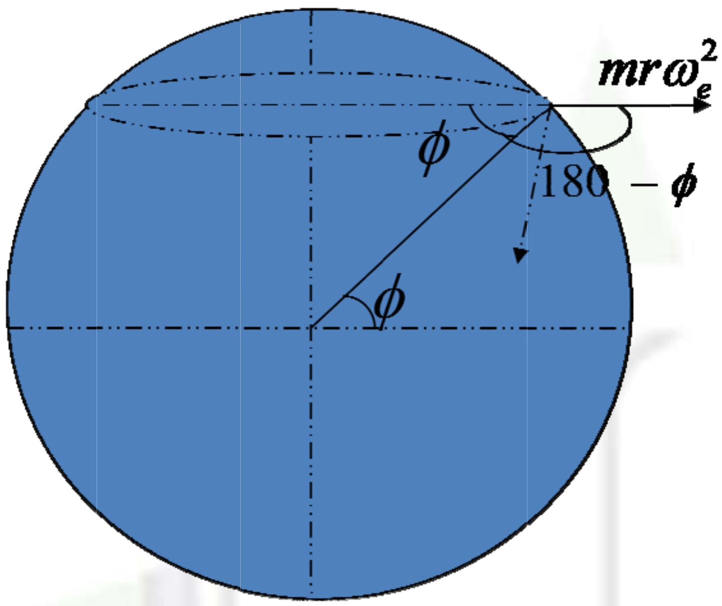
$$g' = G\frac{4}{3}\pi R_e \left(1 - h/R_e\right)$$

$$g' = g\left(1 - h/R_e\right)$$

$g^{'} < g$ thus acceleration due to gravity also decreases below the surface of earth.

(3) Variation of g with latitude:

The angle which a particular place on the surface of earth subtends with the equilateral plane at the center of earth is known as latitude of that place.



From the figure the radius of the circle in which the particle rotates

$$r = R_e \cos\phi$$

$$\omega_e = \frac{2\pi}{T}$$

T = Time taken to make one complete rotation about its own axis = 24 hours

Force acting on the particle

(1) Gravitational force of earth towards the center of earth

(2) The centrifugal force $= \frac{mv^2}{r} = \frac{mr^2\omega_e^2}{r} = mr\omega_e^2 = mR_e \cos\phi\omega_e^2$

Using the law of parallelogram of vectors the resultant force on the particle

$$F = mg^{'} = \sqrt{(mg)^2 + \left(mR_e \cos\phi\omega_e^2\right)^2 + 2mg \times mR_e \cos\phi\omega_e^2 \times \cos(180 - \phi)}$$

$$mg^{'} = mg\sqrt{1 + \frac{R_e^2\omega_e^2 \cos\phi}{g^2} - 2\frac{R_e\omega_e^2 \cos^2\phi}{g}}$$

$$g^{'} = g\left(1 + \frac{R_e^2\omega_e^2 \cos\phi}{g^2} - 2\frac{R_e\omega_e^2 \cos^2\phi}{g}\right)^{1/2}$$

From equation (1) we find that since latitude ϕ is different at different places hence g' will also be different at different places.

At the poles $\phi=\pi/2$, $\cos\phi=\cos\pi/2=0$

$$g' = g$$

Acceleration due to gravity on the surface of earth is maximum at the poles and it is minimum at the equator.