Gravitational field:

When one body exerts force on another from a distance without any actual contact the force is known as field force.

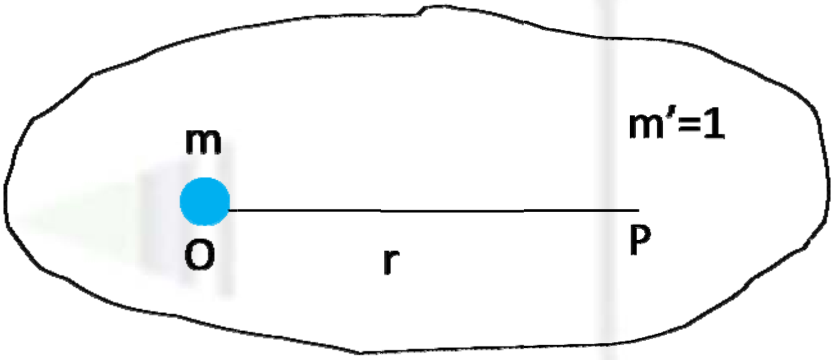
The region surrounding a mass where if another mass is kept it experiences a force then that region is said to be the gravitational field of the given mass.

The strength of the gravitational field at a point can be measured or represented in two different ways.

(1) Gravitational Intensity

(2) Gravitational Potential

Gravitational Intensity: The gravitational intensity at a point in the gravitational field is defined as the force experienced by a unit mass (imaginary) kept at that point.



M = mass of the particle kept at O

P is a point in the gravitational field of a given point mass at O.

Let us imagine a particle of mass m'=1 at P

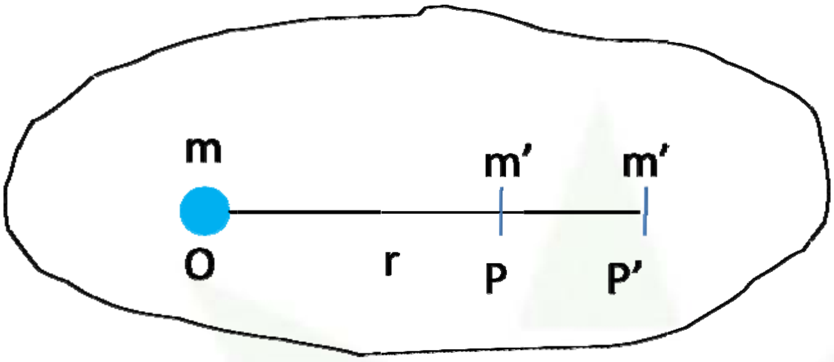
Force experienced by that unit mass at P $= G\frac{m.1}{r^2}$ along PO and this is known as Intensity at P.

$I = G\frac{m}{r^2}$ It is being a force a vector quantity & is always directed toward the given point mass.

Unit of Intensity $= \frac{Force}{mass} = \frac{Newton}{Kg}$

Gravitational Potential:

A point mass m is kept at O' another point mass m' is considered at P in the gravitational field of the mass at O.



The mass m' experiences a force. If we move the mass from P to P' a displacement occurs in presence of force & work is done energy is spent. This work done is stored in that point mass m' at the point P' in the form of energy known as gravitational potential energy.

The reference point from which the particle is to be moved is taken at infinity.

The work done to bring a particle from infinity up to the given point in the gravitational field is stored in that particle in the form of Gravitational Potential Energy at that point.

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"The work done to bring a particle from infinity up to the given point in the gravitational field is stored in that particle in the form of Gravitational Energy at that point."

Potential: The work done to bring a particle of unit mass from infinity up to the given point in the gravitational field is known as Gravitational Potential at that point.

The Gravitational Potential Energy of a particle per unit mass at point is known as Gravitational Potential at that point.

$$\frac{Work}{mass} = \frac{PotentialEnergy}{mass} = Potential$$

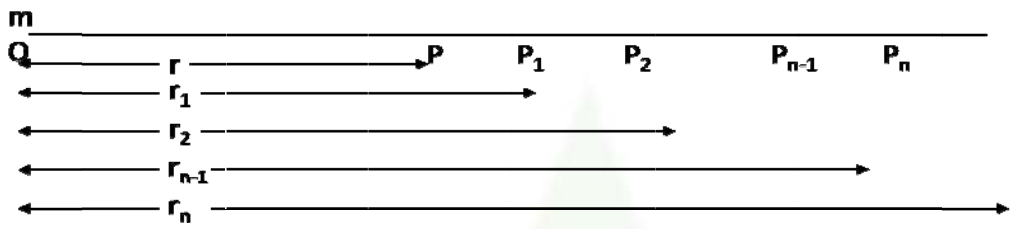
Potential Energy = Potential X mass

Potential Energy: The energy possessed by a body due to its position in a force field is known as potential energy.

Potential energy is calculated from the work done to bring the body from infinity up to the given point in the force field.

Gravitational Potential at a point due to a point mass:

Point mass: If the size of a particle is negligible compared to its distance from the axis (given point) it is known as point mass.



A point mass m is kept at O. P is the given point in the gravitational field of that point mass. Given OP=r, join OP and produce it up to any point P_n ($OP_n = r_n$)

Divide PP_n by points P_1, P_2 …… P_{n-1} at distances r_1, r_2,…… r_{n-1} from O respectively.

Gravitational Force on unit mass kept at P $= G\dfrac{m.1}{r^2}$ along PO

Gravitational Force on unit mass kept at P_1 $= G\dfrac{m}{r_1^2}$ along P_1O

The force changes from P to P_1. Hence the average force on unit mass between the points P & P_1 can be obtained by taking geometric mean of the two forces because the variation in force is due to the inverse square distance.

The average gravitational force on unit mass considered between the points P & P_1

$$= \sqrt{\frac{Gm}{r^2}\frac{Gm}{r_1^2}}$$

$$= \frac{Gm}{r_1 r}$$ Along P_1P

The work done in taking the unit mass from P to P_1 against the gravitational force

= Force on unit mass X distance PP_1

$$= \frac{Gm}{r_1 r}(r_1 - r)$$

$$= Gm\left[\frac{r_1}{rr_1} - \frac{r}{rr_1}\right]$$

$$= Gm\left[\frac{1}{r} - \frac{1}{r_1}\right]$$

$$W_{P\to P_1} = Gm\left[\frac{1}{r} - \frac{1}{r_1}\right]$$

$$W_{P_1\to P_2} = Gm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

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$$W_{Pn-1\to P_n} = Gm\left[\frac{1}{r_{n-1}} - \frac{1}{r_n}\right]$$

Hence the total work done in taking the unit mass from P to P_n can be obtained by taking the algebraic sum of all these work done.

$$W_{P\to P_n} = W_{P_1\to P_2} + W_{P_2\to P_3} + \ldots.. + W_{P_{n-1}\to P_n}$$

$$W_{P\to P_n} = Gm\left[\frac{1}{r} - \frac{1}{r_n}\right]$$

$$W_{P\to\infty} = Gm\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{Gm}{r}$$

Hence the work done to bring the unit mass from infinity up to P in favor of the attractive force can be obtained by putting a negative sign.

$$W_{\infty\to P} = -\frac{Gm}{r}$$

Differentiating equation (2) with respect to r

$$\frac{dv}{dr} = -Gm\frac{d}{dr}\left(\frac{1}{r}\right) = -Gm\left(-\frac{1}{r^2}\right) = \frac{Gm}{r^2} = I$$

The gravitational intensity is rate of change of gravitational potential with distance i.e. potential gradient.