Q18. A radio active nucleus 'A' undergoes a series of decays according to the following scheme:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A are 180 and 72 respectively. What are these numbers for A_4?

Answer : We know that alpha particle when emits atomic mass decreased by 4 unit and atomic number decreased by 2 unit, while Beta emission increases atomic number but does not bring any changes to mass.

Gama emission does not bring any changes neither in atomic mass nor in atomic number.

Alfpha = doubly ionized helium, Beta = electon , Gama = Electro Magnetic wave.

$$ {}^{180}_{72}\mathrm{A} \xrightarrow{\alpha = {}^{4}_{2}\mathrm{He}} {}^{176}_{70}\mathrm{A}_1 \xrightarrow{\beta = {}^{0}_{-1}\mathrm{e}} {}^{176}_{71}\mathrm{A}_2 \xrightarrow{\alpha = {}^{4}_{2}\mathrm{He}} {}^{172}_{69}\mathrm{A}_3 \xrightarrow{\gamma} {}^{172}_{69}\mathrm{A}_4 $$

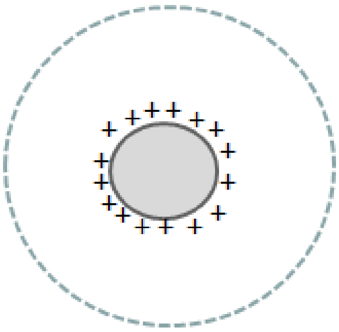
$\therefore$ The mass number of $\mathrm{A}_4 = 172$
and the atomic number of $\mathrm{A}_4 = 69$

Q19. A thin conducting spherical shell of radius R has charge Q spread uniformly over its surface. Using Gauss's law, derive an expression for an electric field at a point outside the shell.

Draw a graph of electric field E(r) with distance r from the centre of the shell for $0 \leq r \leq \infty$.

Applications of Gauss's Theorem :

To find intensity at a point due to a uniformly charged spherical shell



Q= charge on the surface of the shell
R=Radius of the shell
r=OP=distance of the given point from the centre of the shell.
ε=Permittivity of the medium
Let E= Electric intensity at P due to the charged shell.

$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2} \rightarrow (1)$$

Equation (1) is applicable when the given charge is point charge but in this case since the charged shell can not be treated as point charge we can not apply this formula.

Let us imagine a concentric shell of radius r passing through P this is our Gaussian surface.
Surface area of the Gaussian surface $A= 4\pi r^2$
Since the lines of force leave the surface of a charged body perpendicularly hence the lines of force are all radial to the charged sphere and hence force are all radial to the charged sphere and hence they are also radial to the Gaussian surface and cutting the surface perpendicularly.

Using definition of flux through the Gaussian surface $\phi=EA=E.4\pi r^2 \longrightarrow (1)$

Case I : Let the point P lie outside the charged shell OP = r > R

Applying the Gauss's theorem the flux through the Gaussian surface

$$\phi = \frac{\text{Charged enclosed}}{\varepsilon} = \frac{Q}{\varepsilon} \rightarrow (2)$$

Equating (1) and (2):

$$E.4\pi r^2 = \frac{Q}{\varepsilon}$$

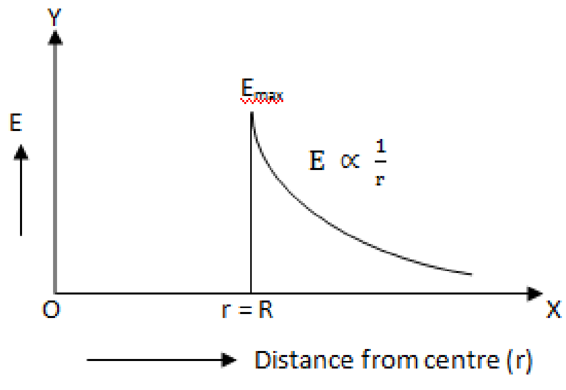
$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2} \rightarrow (3)$$

If we imagine a point charge Q at the center of the shell intensity at P

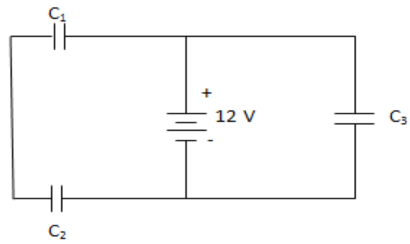
$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2} \rightarrow (4)$$

Thus comparing equation (3) & (4) we can say that to find potential and intensity at any external point due to a charged shell the shell can be assumed to be concentrated at its centre.

Graphical Presentation: The variation of the electric field intensity E(r) with distance r from the centre for shell $0 \leq r \leq \infty$ is shown below:



Q20. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12 V battery as shown.



Find

(i) Charge on each capacitor
(ii) Equivalent capacitance of the network
(iii) Energy stored in the network of capacitors

Ans.

(i) Here $V = 12\text{ V}$ and $C_1 = C_2 = C_3 = 6\mu F = 6 \times 10^{-6} F$
Charge on capacitor C_3 is
$q_3 = C_3 V = 6 \times 10^{-6} \times 12 = 72 \times 10^{-6}$
$= 72\ \mu C$
Since capacitor C_1 and C_3 are in series
$\therefore$ Equivanet capacitance $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}$
$\frac{1}{C_S} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$
$\therefore\ C_S = 3\ \mu F$
Charge on capacitor C_1 and C_2 is
$q = C_S V = 3 \times 10^{-6} \times 12 = 36 \times 10^{-6} = 36\ \mu C$
$\therefore$ Charge on each capacitor C_1 and C_2 is 36 μC

(ii) Since capacitor C_1 and C_3 are in series
$\therefore$ Equivalent capacitance $C_S = 3\ \mu F$
Now C_3 and C_S are in parallel
$\therefore$ Equivalent capacitance $C = C_3 + C_S$
$= 6 + 3 = 9\ \mu F$

(iii) Energy stored
$= \frac{1}{2} CV^2$
$= \frac{1}{2} \times 9 \times 10^{-6} \times (12)^2$
$= \frac{1}{2} \times 9 \times 10^{-6} \times 144 = 648 \times 10^{-6}$
$= 6.48 \times 10^{-4}\ J$