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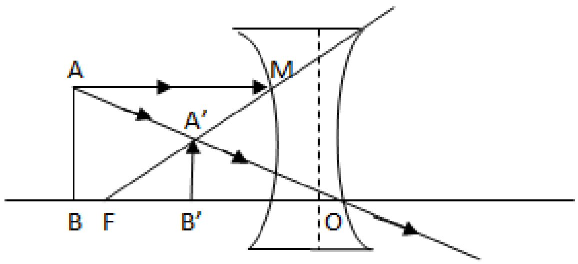
Q29. Derive the lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a concave lens, using the necessary ray diagram.

Two lenses of powers 10 D and -5 D are placed in contact.

(i) Calculate the power of the new lens.

(ii) Where an object should be held from the lens, so as to obtain a virtual image of magnification 2?

Derivation of lens formula: In the figure, AB is the object kept beyond F perpendicular to the principal of a concave lens. A'B' is the erect, virtual and diminished image.



Δ ABO and ΔA′B′O are similar.

$\therefore \frac{AB}{A'B'} = \frac{BO}{B'O} \ldots\ldots\ldots\ldots..(i)$

Δ MOF and ΔA′B′F are similar.

$\frac{MO}{A'B'} = \frac{FO}{FB'}$

But MO = AB

$\frac{AB}{A'B'} = \frac{FO}{FB'} \ldots\ldots\ldots\ldots..(ii)$

From equations (i) and (ii), we get

$\frac{BO}{B'O} = \frac{FO}{FB'}$ or, $\frac{BO}{B'O} = \frac{FO}{FO - B'O}$

Using new Cartesian sign convention,

$BO = -\mu, B'O = -v, FO = -f$

Hence $\frac{-u}{-v} = \frac{-f}{-f+v}$

Or, uf – uv = vf

Or, uv = uf – vf

Dividing both sides by uvf, we have

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

This is the required formula for a thin concave lens.

(i) Power of new lens, $P = P_1 + P_2$

$\therefore P = 10 - 5 = +5D$

(ii) Here, $u = ?, f = \frac{1}{P} = \frac{100}{5} = 20$ cm

$m = \frac{v}{u} \Rightarrow 2 = \frac{-v}{-u} \quad \therefore v = 2u$

Using lens formula

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{20} = \frac{1}{2u} - \frac{1}{u}$

$\frac{1}{20} = \frac{1-2}{2u} \Rightarrow \frac{1}{20} = \frac{-1}{2u} \quad \therefore u = -10$ cm.

$\therefore$ Object distance $= 10$ cm.

Q29 Part II

(a) What are the coherent sources of light? Two slits in Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. Why is no interference pattern observed?
(b) Obtain the condition for getting dark and bright fringes in Young's experiment. Hence write the expression for the fringe width.
(c) If s is the size of the source and its distance from the plane of the two slits, what should be the criterion for the interference fringes to be seen?

Answer

(a) **Coherent sources of light:** The sources of light, which emits continuous light waves of the same wavelength, same frequency and in same phase are called **Coherent sources of light.** Coherent sources are always produced from a single source in two different ways.

(1) By division of wave front : We know any point laying on the wave front act as secondary coherent sources. We have to expose only two such points blocking all other points as done in Young's experiment.

(2) By division of amplitude : We know that intensity of light is proportional to square of the amplitude.

Interference pattern is not obtained. This is because phase difference between the light waves emitted from two different sodium lamps will change continuously.

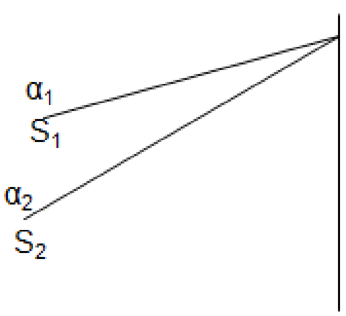
(b)

Let d = distance of separation between two slits.

D = Perpendicular distance of the screen from the slits

a & R = amplitude of the waves emitted from S_1 and S_2 respectively.

λ = wave length of light ($\omega = 2\pi v = 2\pi c/\lambda$)



$$\beta_1 = \alpha_1 + \frac{2\pi}{\lambda}x_1$$

$$\beta_2 = \alpha_2 + \frac{2\pi}{\lambda}x_2$$

$$\phi = \beta_2 - \beta_1 = \alpha_2 + \frac{2\pi}{\lambda}x_2 - \left(\alpha_1 + \frac{2\pi}{\lambda}x_1\right)$$

$$\phi = (\alpha_2 - \alpha_1) + \frac{2\pi}{\lambda}(x_2 - x_1)$$

Let y_1 and y_2 be the displacement at any point P on the screen at an instant of time t of the waves emitted from S_1 and S_2 respectively. Using displacement equation:

$$y_1 = a \sin \omega t \rightarrow (1)$$

$$y_2 = a \sin (\omega t \pm \phi) \rightarrow (2)$$

$$\phi = (\alpha_2 - \alpha_1) + \frac{2\pi}{\lambda}(x_2 - x_1) \rightarrow (3)$$

$$y = y_1 + y_2 = a \sin \omega t + a \sin (\omega t \pm \phi)$$

$$y = a \sin \omega t + b \sin \omega t . \cos \phi \pm b \cos \omega t . \sin \phi$$

$$y = (a + b \cos \phi) \sin \omega t \pm b \cos \omega t . \sin \phi \rightarrow (4)$$

$$\textit{Putting} \quad \text{b} \sin \phi = \text{c} \sin \delta \rightarrow (5)$$

$$\text{a} + \text{b} \cos \phi = \text{c} \cos \delta \rightarrow (6)$$

Putting equation (5) and (6) in (4)

$$\text{y} = \text{c} \sin \omega \text{t} . \cos \delta \pm \text{c} \cos \omega \text{t} . \sin \delta$$

$$\text{y} = \text{c} \sin (\omega \text{t} \pm \delta) \rightarrow (7)$$

From equation (7) we find that c is the amplitude of the resultant wave due to the superposition. Squaring and adding equation (5) & (6)

$$c^2 = b^2\sin^2\phi + a^2 + b^2\cos^2\phi + 2ab\cos\phi$$
$$c^2 = a^2 + b^2 + 2ab\cos\phi \rightarrow (8)$$

I = Resultant intensity at P due to the superposition of the two waves

$$I \propto (amplitude)^2$$
$$I \propto c^2$$
$$I = \text{constant}.c^2$$
$$I = \text{constant}.\left[a^2 + b^2 + 2ab\cos\phi\right] \rightarrow (9)$$

From equation (9) we find I is maximum when $\cos\phi$ is maximum.

$$I = \text{constant}\left[a^2 + b^2 + 2ab\cos\phi\right]$$
$$\cos\phi = 1 = \cos 0 = \cos 2\pi = \cos 4\pi = \cos 6\pi = \ldots$$
$$\phi = 2n\pi \rightarrow (10) \text{ where } n = 0,1,2,3,4,\ldots$$
$$\text{or } \frac{2\pi}{\lambda}\Delta = 2n\pi \rightarrow (11)$$
$$\Delta = \frac{2n\pi}{2\pi}\lambda = 2n\frac{\lambda}{2} \rightarrow (12)$$

where n = 0,1,2,3,... an integer

$$\Delta = 0, \lambda, 2\lambda, 3\lambda, 4\lambda \ldots\ldots$$

Condition of minimum intensity (Destructive interference): From equation (9) intensity is minimum when $\cos\phi$ is minimum i.e. $\cos\phi = -1 = \cos\pi = \cos 2\pi = \cos 3\pi, \ldots$

i.e. $\phi = \pi, 3\pi, 5\pi, 7\pi, \ldots$

$\phi = (2n+1)\pi \rightarrow$ (13)

where n=0,1,2,3,4,..

$$\therefore \frac{2\pi}{\lambda}\Delta = (2n+1)\pi$$

or $\Delta = (2n+1)\frac{\lambda}{2} \rightarrow (14)$

Where n = 0,1,2,3,... .. an integer

i.e. $\Delta = 1.\frac{\lambda}{2},\ 3\frac{\lambda}{2},\ 5\frac{\lambda}{2},\ 7\frac{\lambda}{2},\dots\dots\dots$

Fringe width : The distance of separation between any two consecutive bright or dark fringes is known as fringe width.

The constructive interference happen when wavelets emitting from both sources are in phase

i.e $S_2P - S_1P = n\lambda; n = 0, 1, 2\dots$

Then

$$S_2P = \left(D^2 + \left(x + \frac{d}{2}\right)^2\right)^{\frac{1}{2}} \approx D\left[1 + \frac{1}{2}\left(\frac{x + \frac{d}{2}}{D}\right)^2\right]$$

$$S_1P = D\left[1 + \frac{1}{2}\left(\frac{x - \frac{d}{2}}{D}\right)^2\right]$$

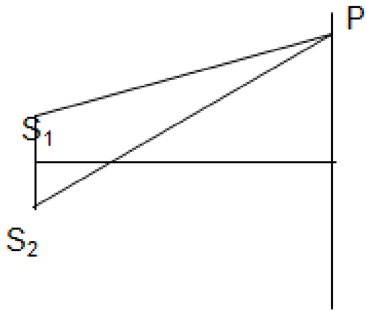
Therefore, path difference

$$S_2P - S_1P = \frac{\left(x + \frac{d}{2}\right)^2 - \left(x - \frac{d}{2}\right)^2}{2D} = \frac{xd}{D}$$

Therefore, the point P will be maximum intensity if

$$\frac{xd}{D} = n\lambda$$

or $$x = \frac{n\lambda D}{d}$$



The condition for destructive interference is

$$\text{path difference} = \left(n + \frac{1}{2}\right)\lambda$$

Therefore,

$$\frac{xd}{D} = \left(n + \frac{1}{2}\right)\lambda$$

or $$x = \frac{(2n+1)D\lambda}{2d}$$

The fringe width is the separation between the consecutive bright or dark fringes

Taking bright fringes

$$\beta = x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$$

(c) The conditions for interference fringes to be seen is

(1)

$$I_{max} = \text{constant } \left[a^2 + b^2 + 2ab\right]$$
$$I_{max} = \text{constant } (a + b)^2 \rightarrow (15)$$
$$I_{min} = \text{constant } \left[a^2 + b^2 + 2ab(-1)\right]$$
$$I_{min} = \text{constant } (a - b)^2 \rightarrow (16)$$

If a=b i.e. if the amplitude of the two interfering waves are equal then from equation (15) and (16) : I_{max}=constant.$4a^2$, I_{min}= 0. Contrast between the bright and dark fringe is very high and the fringes are very good.

If a>>b then b can be neglected compared to a and hence from (15) and (16) I_{max}=constant.a^2 ,

I_{min}= constant.a^2 so fringes can not be seen.

Thus to get good interference fringes the amplitude of the two interfering waves must be either exactly equal or very nearly equal. But if they are widely different no fringes can be seen.

(2)

D=1 m =102 cm

λ =60000A=6 x 10-5 cm. then Fringe width

$$\bar{x} = 0.006\ cm$$

We find that the fringe width is so small that bright and dark fringes will overlap. Hence to have distinct interference fringes the distance of separation between the two coherent sources should be very small, smaller than 1mm.

(3) From Equation 9 we find that resultant intensity

$$I = \text{constant}.\left[a^2 + b^2 + 2ab\cos\phi\right] \rightarrow (9)$$

$$\phi = (\alpha_2 - \alpha_1) + \frac{2\pi}{\lambda}(x_2 - x_1)$$

If the sources are not coherent the initial phase difference (α_2- α_2) changes with time 10^8 times per second, i.e. intensity at every point changes 10^8 times per second. Since human eye can not follow such quick change hence intensity at every point appears to be same i.e. no fringes can be seen. But if the sources are coherent initial phase are same α_2- α_2 =0 . path difference is different at different points on the screen hence the phase difference Ø and so the intensity I are different at different points on the screen and we get interference fringes.

Q30. An a.c. source generating a voltage $v = v_m \sin \omega t$ is connected to a capacitor of capacitance C. Find the expression for the current, I, flowing through it. Plot a graph of v and I versus ωt to show that the current is $\pi/2$ ahead of the voltage.

A resistor of $200\ \Omega$ and a capacitor of $15.0\ \mu F$ are connected in series to a 220 V, 50 Hz a.c. source. Calculate the current in the circuit and the rms voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Ans. **A.C. source containing capacitor:** Let a source of alternating emf $V = V_m \sin \omega t$ be connected to a capacitor of capacitance C only.

$V = V_m \sin \omega t \ \ldots\ldots\ldots (1)$

At every instant, the potential V is given by

$V = \frac{q}{c} \Rightarrow V_m \sin \omega t$

$\therefore q = C\, V_m \sin \omega t$

If i is instantaneous value of current in the circuit at instant t, then

$i = \frac{dq}{dt} = \frac{d}{dt}(C\, V_m \sin \omega t)$

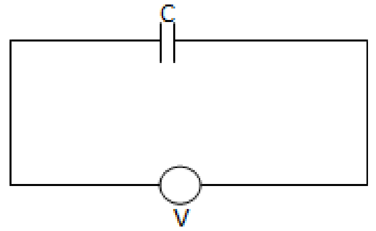
$i = C\, V_m (\cos \omega t).\omega = \frac{V_m}{1/\omega c} \sin\left(\omega t + \frac{\pi}{2}\right)$

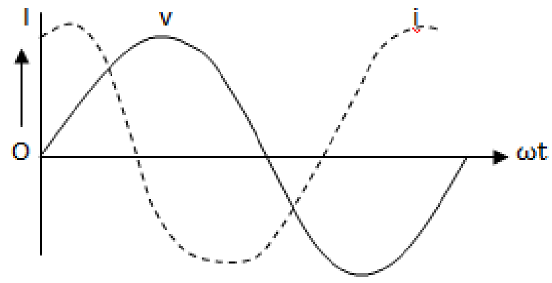
The current will be maximum when $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$

$\therefore\ i_m = \frac{V_m}{1/\omega c} \times 1 = \frac{V_m}{1/\omega c}$

$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$

Therefore, alternating current I lead the alternating voltage by a phase angle of $\frac{\pi}{2}$.





Given $R = 200\ \Omega, C = 15.0\ \mu F, = 15 \times 10^{-6}$ F,

$V_{rms} = 220$ V, $\nu = 50$ Hz, $I_{rms} = ?$

$X_c = \frac{1}{\omega c} = \frac{1}{2\pi\nu c}$

$= \frac{1}{2\times\frac{22}{7}\times 50\times 15\times 10^{-6}} = \frac{7\times 10^6}{33000}$

$= 212.12 = 212\ \Omega$

$\therefore Z = \sqrt{R^2 + {X_c}^2} = \sqrt{200^2 + 212^2}$

$= \sqrt{40000 + 44944} = \sqrt{84944} = 291.45\ \Omega$

$\therefore\ I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{291.45} = 0.75$ A

$\therefore\ V_R = I_{rms} R = 0.75 \times 200 = 150$ V

$V_C = I_{rms} . X_c = 0.75 \times 212 = 159$ V

$\therefore\ V_R + V_C = 150 + 159 = 309$

$\therefore\ V_R + V_C > V$

This (paradox) is because these voltages **are not in same phase** and they can not be added like ordinary numbers.

$\therefore V = \sqrt{V_R^2 + V_C^2}$

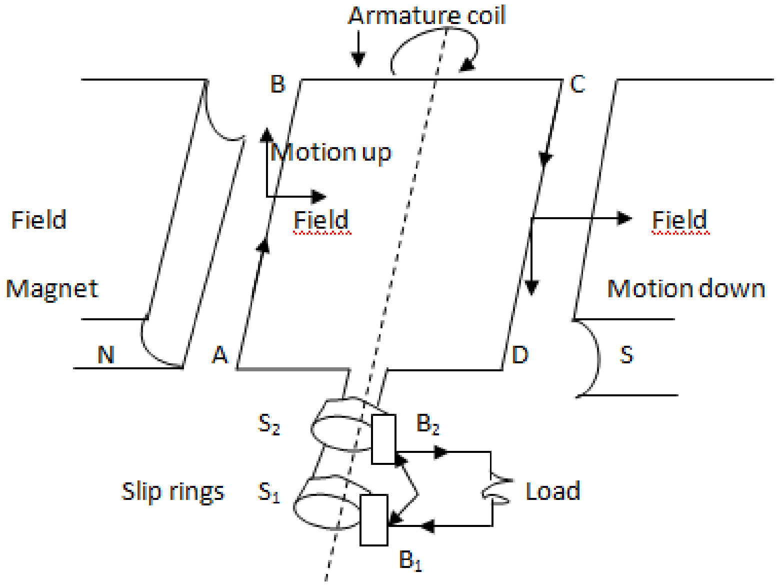
$= \sqrt{150^2 + 159^2} = \sqrt{22500 + 25281}$

$= \sqrt{47781} = 218.58\text{ V}$

Or,

Explain briefly, with the help of a labeled diagram, the basic principle of the working of an a.c. generator.
In an a.c. generator, coil of N turns and area A is rotated at v revolutions per second in a uniform magnetic field B. write the expression for the emf produced.
A 100-turn coil of area 0.1 m^2 rotates at half a revolution per second. It is placed in a magnetic field 0.01 T perpendicular to the axis of rotation of the coil. Calculate the maximum voltage generated in the coil.

***Principle Of* A. C. Generator :** Whenever a closed coil is rotated in a uniform magnetic field about an axis perpendicular to the field, the magnetic flux linked with coil changes and an induced emf is set up across its ends.



The essential parts of an a.c. generator are shown in the figure. Initially the armature coil ABCD is horizontal. As the coil is rotated clockwise, the arm AB moves up and CD moves down. By Fleming's right hand rule, the induced current flows along ABCD. In second half rotation, the arm CD moves up and AB moves down. The induced current flows in the opposite direction i.e., along DCBA. Thus an alternating current flows in the circuit.

The magnetic flux linked with the coil at any instant is

$\phi = \text{NB A} \cos \omega t$

Induced emf will be

$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(\text{NB A } \cos \omega t)$

$= \text{NBA } \omega \sin \omega t$

or, $E = E_0 \sin \omega t$

Where $E_0 = \text{NBA } \omega$ = peak value of induced emf.

Solution of Numerical Problem: Given N= 100, A = 0.1 m^2, B = 0.01 T

$\nu = \frac{1}{2}$ revolution per sec = 0.5 r. p. s

∴ Maximum voltage generated $e_0 = \text{NBA } \omega = \text{NBA } (2\pi \nu)$

$\therefore e_0 = 100 \times 0.01 \times 0.1 \times 2 \times \frac{22}{7} \times 0.5$

$= \frac{2.2}{7} = 0.314$ Volt

$e_{rms} = \frac{e_0}{\sqrt{2}} = \frac{0.314}{1.414} = 0.22$ V .