12. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1)
(1) does not meet the curve again (2) meets the curve again in the second quadrant
(3) meets the curve again in the third quadrant. (4) meets the curve again in the fourth quadrant.

Answer:
$x^2 + 2xy - 3y^2 = 0 \rightarrow (1)$

Differentiating with respect to y
$$2x + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$$
$$or\ 2(x - 3y)\frac{dy}{dx} + 2(x + y) = 0$$
$$or\ \frac{dy}{dx} = -\frac{x + y}{x - 3y}$$
$$or\ \left(\frac{dy}{dx}\right)_{1,1} = -\frac{2}{-2} = 1$$
Therefore equation of normal at (1, 1)
$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}(x - x_1)$$
$$or\ y - 1 = -1(x - 1)$$
$$or\ y - 1 = -x + 1$$
$$or\ x + y = 2 \rightarrow (2)\ Equation\ of\ normal\ \ Since\ it\ cuts\ the\ curve$$
$$puting\ y = 2 - x\ in\ equation\ (1)\ we\ get$$
$$x^2 + 2x(2 - x) - 3(2 - x)^2 = 0$$
$$or\ x^2 + 4x - 2x^2 - 3(4 + x^2 - 4x) = 0$$
$$or - 4x^2 + 16x - 12 = 0\ or\ x^2 - 4x + 3 = 0$$
$$or\ x^2 - 3x - x + 3 = 0$$
$$or\ x(x - 3) - 1(x - 3) = 0$$
$$or (x - 3)(x - 1) = 0$$
$$or\ x = 1,3\ when\ x = 1, y = 1\ when\ x = 3, y = -1$$
$$So\ the\ normal\ meets\ the\ curve\ at\ A(1,1) and\ B(3, -1).$$
$$A(1,1) given\ in\ Question\ 2nd\ point\ B(3, -1) is\ in\ 4th\ Quadrant$$

Correct option (4) meets the curve again in the fourth quadrant.