



15. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to

(1) 2

(2) 4

(3) 1

(4) 6

Answer:

$$\text{Let } I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$$

$$\text{or } I = \int_2^4 \frac{2\log x}{2\log x + \log(6 - x)^2} dx$$

$$\text{or } I = \int_2^4 \frac{2\log x}{2\log x + 2\log(6 - x)} dx$$

$$I = \int_2^4 \frac{\log x}{\log x + \log(6 - x)} dx \rightarrow (1)$$

From the properties of definite integral we know that

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

Therefore

$$I = \int_2^4 \frac{\log(6 - x)}{\log(6 - x) + \log[6 - (6 - x)]} dx$$

$$I = \int_2^4 \frac{\log(6 - x)}{\log(6 - x) + \log x} dx \rightarrow (2)$$

Adding (1) and (2) we get

$$2I = \int_2^4 \frac{\log x + \log(6 - x)}{\log(6 - x) + \log x} dx$$

$$\text{or } 2I = [x]_2^4$$

$$\text{or } I = 1$$

Correct option is (3) 1