SelfStudy.in

2. A complex number z is said to be unimodular if |z| = 1. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\overline{z_2}}$ is unimodular and z_2 is not unimodular.
Then the point z_1 lies on a:

(1) straight line parallel to x-axis
(2) straight line parallel to y-axis
(3) circle of radius 2
(4) circle of radius $\sqrt{2}$

Answer:

Given z is unimoduler i.e. |z| = 1, therefore $|z|^2 = z\bar{z}$ we know that multiplying complex number by its conjugate it remains same. Therefore multiplying $\frac{z_1-2z_2}{2-z_1\overline{z_2}}$ by its conjugate

$$\left(\frac{z_1 - 2z_2}{2 - z_1\overline{z_2}}\right)\left(\frac{\overline{z_1} - 2\overline{z_2}}{2 - \overline{z_1}z_2}\right) = 1$$

$$or\ |z_1|^2 - 2z_1\overline{z_2} - 2z_2\overline{z_1} + 4z_2\overline{z_2} = 4 - 2\overline{z_1}z_2 - 2z_1\overline{z_2} + |z_1|^2|z_2|^2$$

$$or\ |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0$$

$$or\ \ |z_1|^2(1 - |z_2|^2) - 4(1 - |z_2|^2) = 0$$

$$or(1 - |z_2|^2)(|z_1|^2 - 4) = 0$$

$$since\ z_2\ is\ not\ unimoduler\ therefore\ |z_2|^2 \neq 1$$

$$hence\ \ |z_1| = 2\ , i.e. z_1\ lies\ in\ a\ circle\ of\ radius\ 2.$$

Correct option is (3)