21. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is

(1) $\frac{27}{4}$ (2) 18 (3) $\frac{27}{2}$ (4) 27

Answer:

Area=$2\frac{a^2}{e}=\frac{2a^2}{\sqrt{1-\frac{b^2}{a^2}}}=\frac{2\times 9}{\sqrt{1-\frac{5}{9}}}=\frac{18\times 3}{2}=27$

Alternative detail method:

We know that latus rectum to the ellipse passes through focus co-ordinate (ae, 0) laying on major axis.

$$\frac{x^2}{9}+\frac{y^2}{5}=1, a^2=9, b^2=5,$$

$$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{5}{9}}=\frac{2}{3},$$

Finding co-ordinate $(x_1,y_1)$of the point of contact of tangent and latus rectum on the ellipse at first quadrant,

$$x_1=ae=3\times\frac{2}{3}=2$$

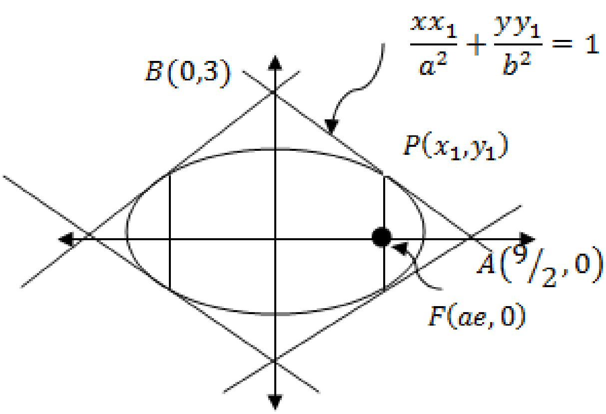
$(\,latus\ rectum\ passing\ through\ focus).$

$Here\ x_1\ lies\ on\ \frac{x^2}{9}+\frac{y^2}{5}=1$

$hence\ y_1\ can\ be\ found\ as$

$\frac{4}{9}+\frac{{y_1}^2}{5}=1$

$or\ {y_1}^2=5\times\left(1-\frac{4}{9}\right)=\frac{25}{9}, y_1=\frac{5}{3}$



Therefore equation of tangent to ellipse at $(x_1,y_1)=\left(2,\frac{5}{3}\right)$

$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

$or\ \frac{x\times 2}{9}+\frac{y\times\frac{5}{3}}{5}=1$

$or\ \frac{x}{9/2}+\frac{y}{3}=1,$

$hence\ X-axis\ intercept\ A(^9/_2,0), Y\ axis\ intercept\ of\ tangent\ B(0,3)$

Therefore Area of the triangle formed by co-ordinate axes and tangent=$\frac{1}{2}\times\frac{9}{2}\times 3=\frac{27}{4}$

Therefore total area of the quadrilateral $=4\times\frac{27}{4}=27$

Correct option is (4) 27