25. Let $\vec{a}, \overrightarrow{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of sinθ is

(1) $\frac{2\sqrt{2}}{3}$ (2)- $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4)- $\frac{2\sqrt{3}}{3}$

Answer:

$$(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$or -\vec{c} \times (\vec{a} \times \vec{b}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$or - (\overrightarrow{\vec{c}.b})\vec{a} + (\overrightarrow{\vec{c}.\vec{a}}\,)\vec{b} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$or\ (\vec{c}.\vec{a}\,)\overrightarrow{b} = \left(\frac{1}{3}|\vec{b}||\vec{c}| + \vec{c}\vec{b}\right)\vec{a}$$

$$Since\ \vec{a}\ and\ \overrightarrow{b}\ are\ non\ colliner\ therefore\ \ \overrightarrow{b}\ can\ not\ be\ written\ as\ sclar\ multiple\ of\ \vec{a}$$

$$\left(\frac{1}{3}|\vec{b}||\vec{c}| + \overrightarrow{\vec{c}.b}\right) = 0\ and\ \vec{c}.\vec{a} = 0$$

$$Therefore\ \frac{1}{3}bc + bccos\theta = 0$$

$$or\ cos\theta = -\frac{1}{3}$$

$$hence\ sin\theta = \sqrt{1 - cos^2\theta} = \frac{2\sqrt{2}}{3}$$

Correct choice is (1) $\frac{2\sqrt{2}}{3}$