



29. Let  $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ , where  $|x| < \frac{1}{\sqrt{3}}$ . Then a value of  $y$  is

- (1)  $\frac{3x-x^3}{1-3x^2}$       (2)  $\frac{3x+x^3}{1-3x^2}$       (3)  $\frac{3x-x^3}{1+3x^2}$       (4)  $\frac{3x+x^3}{1+3x^2}$

**Answer:**

Let  $\tan^{-1}x = \theta$ ,  $x = \tan\theta$ , Since  $x < \frac{1}{\sqrt{3}}$ ,  $\tan\theta < \tan\frac{\pi}{6}$ ,  $\theta < \frac{\pi}{6}$

$$\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\text{or } \tan^{-1}y = \theta + \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$\text{or } \tan^{-1}y = \theta + \tan^{-1}(\tan 2\theta)$$

$$\text{or } \tan^{-1}y = \theta + 2\theta$$

$$\text{or } \tan^{-1}y = 3\theta$$

$$\text{or } y = \tan 3\theta$$

$$\text{or } y = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$\text{or } y = \frac{3x - x^3}{1 - 3x^2}$$

**Correct option is (1)  $\frac{3x-x^3}{1-3x^2}$**