



3. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to:

(1) 6

(2) -6

(3) 3

(4) -3

Answer:

$x^2 - 6x - 2 = 0$ here $\alpha + \beta = 6$ and $\alpha \times \beta = -2$

Given $a_n = \alpha^n - \beta^n$ hence $\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} \rightarrow (1)$

Since α, β are roots therefore

$\alpha^2 - 6\alpha - 2 = 0$ or $\alpha^2 - 2 = 6\alpha$

similarly

$\beta^2 - 6\beta - 2 = 0$ or $\beta^2 - 2 = 6\beta$

Putting in equation (1) we get

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^8 6\alpha - \beta^8 6\beta}{2(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{2(\alpha^9 - \beta^9)} = 3$$

Correct answer is option (3) 3