



4. If  $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$  is a matrix satisfying the equation  $AA^T = 9I$ , where  $I$  is  $3 \times 3$  identity matrix, then the ordered pair  $(a, b)$  is equal to:

(1)  $(2, -1)$

(2)  $(-2, 1)$

(3)  $(2, 1)$

(4)  $(-2, -1)$

**Answer:**

Given  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$

$A^T = \text{Transpose of } A \text{ obtained by converting rows to column} = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$

$I = \text{Identity matrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$AA^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or 1st row of first matrix and with 3rd column 2nd matrix multiplication gives

$a + 4 + 2b = 0 \rightarrow (1)$  also 2nd row of first with 3rd of 2nd on multiplication gives

$2a + 2 - 2b = 0 \rightarrow (2)$

from (1) and (2)  $3a + 6 = 0$

or  $a = -2$  putting in (2)  $b = -1$  so  $(a, b) = (-2, -1)$

**Hence correct choice is option (4)  $(-2, -1)$**