5. The set of all value of λ for which the system of linear equations:

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

Has a non-trivial solution,

(1) is an empty set

(2) is a singleton

(3) contains two elements

(4) contains more than two elements

Answer: Set of linear equations have non-trivial solution or not depends on the determinant of A. If $|A| = 0$, then we cannot find the inverse of A, so there may exists non-trivial solutions.

$x_1(2-\lambda) - 2x_2 + x_3 = 0$

$2x_1 - x_2(3+\lambda) + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

$$|A| = \begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$$or\ (2-\lambda)\{(-\lambda)(-3-\lambda) - 4\} - 2\{(-\lambda)(-2) - 2\} + (-1)\{-4 + 3 + \lambda) = 0$$

$or\ (2-\lambda)(\lambda^2 + 3\lambda - 4) - 2(2\lambda - 2) + 1 - \lambda = 0$

$or\ 2\lambda^2 + 6\lambda - 8 - \lambda^3 - 3\lambda^2 + 4\lambda - 4\lambda + 4 + 1 - \lambda = 0$

$or\ -\lambda^3 - \lambda^2 + 5\lambda - 3 = 0$

$or\ \lambda^3 + \lambda^2 - 5\lambda + 3 = 0$

$or\ \lambda^3 - \lambda + \lambda^2 - \lambda + 3 - 3\lambda = 0$

$or\ \lambda(\lambda^2 - 1) + \lambda(\lambda - 1) - 3(\lambda - 1) = 0$

$or\ (\lambda - 1)(\lambda^2 + \lambda + \lambda - 3) = 0$

$or\ (\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0$

$or\ (\lambda - 1)(\lambda^2 + 3\lambda - \lambda - 3) = 0$

$or\ (\lambda - 1)\{\lambda(\lambda + 3) - 1(\lambda + 3)\} = 0$

$or\ (\lambda - 1)\{\lambda(\lambda + 3) - 1(\lambda + 3)\} = 0$

$or\ (\lambda - 1)(\lambda + 3)(\lambda - 1) = 0$

$or\ \lambda = 1,3$

Correct choice is option (3) contains two elements