8. If m is the A. M. of two distinct real numbers *l* and n (*l*, n > 1) and G_1, G_2 and G_3 are three geometric means between *l* and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals:

(1) 4 l^2 mm (2) 4 lm^2 n (3) 4 lm n^2 (4) 4 $l^2m^2n^2$

Answer:

Given: $l, m, n\ in\ A.P\ therefore\ m = \frac{l+n}{2} \rightarrow (1)$

$l, G_1, G_2, G_3, n\ are\ in\ G.P$

Let r be the common ratio $n = lr^4\ or\ r = \left(\frac{n}{l}\right)^{\frac{1}{4}}$

$$G_1 = lr = l\left(\frac{n}{l}\right)^{\frac{1}{4}}, {G_1}^4 = nl^3$$

$$G_2 = lr^2 = l\left(\frac{n}{l}\right)^{\frac{2}{4}}, {G_2}^4 = n^2l^2$$

$$G_3 = lr^3 = l\left(\frac{n}{l}\right)^{\frac{3}{4}}, {G_3}^4 = n^3l$$

Therefore

$$G_1^4 + 2G_2^4 + G_3^4 = nl^3 + 2n^2l^2 + n^3l = nl(l^2 + 2nl + n^2) = nl(n+l)^2 = nl(2m)^2 = 4lm^2n$$

Correct option is (2) n^3l