9. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots.$ is

(1) 71 (2) 96 (3) 142 (4) 192

Answer:

The given series is sum of terms of ratio whose numerator is of sum of cube of numbers also 2nd term contains 2 terms 3rd term contains 3 numbers; denominator is sum of odd n natural numbers.

Hence nth term can be written as

$$t_n=\frac{1^3+2^3+\cdots+n^3}{1+3+\cdots+n}$$

$$t_n=\frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\frac{n[2\times 1+(n-1)\times 2]}{2}}$$

$$t_n=\frac{\frac{n^2(n+1)^2}{4}}{n^2}=\frac{n^2+2n+1}{4}$$

$$S_n=\sum_{n=1}^{n}t_n=\frac{\left[\frac{n(n+1)(2n+1)}{6}+n(n+1)+n\right]}{4}$$

$$or\ S_9=\frac{1}{4}\left[\frac{9\times 10\times 19}{6}+90+9\right]$$

$$or\ S_9=\frac{1}{4}[384]=96$$

Correct choice is (2) 96