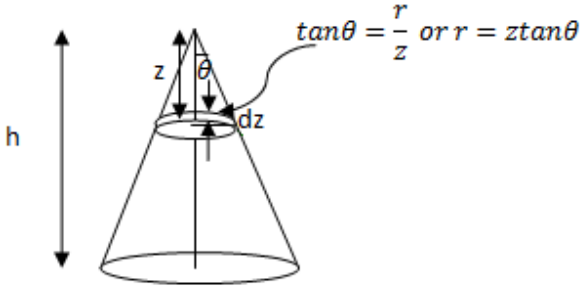
5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to

(1) $\frac{h^2}{4R}$ (2) $\frac{3h}{4}$ (3) $\frac{5h}{8}$ (4) $\frac{3h^2}{8R}$

Answer:



Considering an elementary disc of cone of thickness dz at a distance z from vertex.
Volume of the disc = $\pi(ztan\theta)^2 dz$
Mass of the disc =$dm = \rho\pi(ztan\theta)^2 dz$
We know that Centre of Mass

$$Z_{cm} = \frac{\int_0^h z dm}{\int_0^h dm}$$

$$= \frac{\int_0^h z\rho\pi(ztan\theta)^2 dz}{\int_0^z \rho\pi(ztan\theta)^2 dz}$$

$$= \frac{\int_0^h z^3 dz}{\int_0^h z^2 dz}$$

$$= \frac{h^4}{4 \times \frac{h^3}{3}} = \frac{3}{4}h$$

Therefore centre of mass is at a distance $\frac{3}{4}h$ from the vertex, correct answer is **(2)** $\boldsymbol{\frac{3}{4}h}$