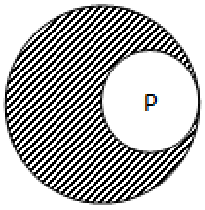
7. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞ the potential at the centre of the cavity thus formed is: (G = gravitational constant)




1. $\frac{-GM}{2R}$  2. $\frac{-GM}{R}$  3. $\frac{-2GM}{3R}$  4. $\frac{-2GM}{R}$

**Answer**:

We know that Gravitational potential at a distance r for the solid sphere of radius a is given by

$$V = \frac{-GM}{2a^3}(3a^2 - r^2) \rightarrow (1)$$

Using equation (1) we can find potential at P due to solid sphere :

$$V_2 = \frac{-GM\left(3R^2 - \left(\frac{R}{2}\right)^2\right)}{2R^3}$$

$$= \frac{-GMR^2\left(\frac{11R^2}{4}\right)}{2R^3} = \frac{-11GM}{8R} \rightarrow (2)$$

$$Density(\rho) = \frac{M}{\frac{4\pi R^3}{3}} = \frac{3M}{4\pi R^3}$$

Therefore Mass of the sphere (cut section) of radius R/2 $= \frac{3M}{4\pi R^3} \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$

Potential at P due to this cut section :

$$V_2 = -\frac{3G \times Mass}{2XRadius} = \frac{-3G\frac{M}{8}}{2\frac{R}{2}} = \frac{-3GM}{8R} \rightarrow (2)$$

Therefore Net Potential =

$$V = V_1 - V_2 = \frac{-11GM}{8R} - \left(\frac{-3GM}{8R}\right)$$

$$or\ V = -\frac{GM}{R}$$

Correct Answer is option (2) $-\frac{GM}{R}$