8. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M ,If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal (g = gravitational acceleration)

1. $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ 2. $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$ 3. $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ 4. $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

Answer:

We know that $T = 2\pi\sqrt{\frac{l}{g}}$

Due to additional mass assuming length of string changed to l_1

Therefore $T_M = 2\pi\sqrt{\frac{l_1}{g}}$

$$\frac{T}{T_M} = \sqrt{\frac{\frac{l}{g}}{\frac{l_1}{g}}} \; or \; \frac{T^2}{{T_M}^2} = \frac{l}{l_1}$$

$$or \; \frac{{T_M}^2}{T^2} - 1 = \frac{l_1}{l} - 1$$

$$or \left[\left(\frac{T_M}{T}\right)^2 - 1\right] = \frac{\Delta l}{l} \rightarrow (1)$$

We know that Young modulus is given by

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{mg}{A} \times \frac{l}{\Delta l}$$

$$or \; \frac{1}{Y} = \frac{A \times \Delta l}{mg \times l} \rightarrow (2)$$

Putting value of $\frac{\Delta l}{l}$ from equation (1) we get

$$\frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{mg}$$

Correct choice is option (1) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{mg}$