9. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $P = \frac{1}{3}\left(\frac{U}{V}\right)$.If the shell now undergoes an adiabatic expansion the relation between T and R is

(1) $T \propto e^{-R}$ (2) $T \propto e^{-3R}$ (3) $T \propto \frac{1}{R}$ (4) $T \propto \frac{1}{R^3}$

Answer: Given $\frac{U}{V} \propto T^4 \; and \; P = \frac{1}{3}\left(\frac{U}{V}\right)$

We know that $PV = nRT \; or \; P = \frac{nRT}{V}$

$$thefore \; \frac{nRT}{V} = \frac{1}{3}\left(\frac{U}{V}\right)$$

$$or \; \frac{nRT}{V} \propto \frac{1}{3}T^4$$

$$or \; nR \propto \frac{1}{3}VT^3$$

$$or \; VT^3 = constant.$$

Volume of spherical shell containing ideal gas $V = \frac{4}{3}\pi R^3$

$$or \; \frac{4}{3}\pi R^3 T^3 = constant$$

$$or \; RT = constant$$

$$or \; T \propto \frac{1}{R}$$

Correct option is (3) $T \propto \frac{1}{R}$