

JEE Advanced 2015 Physics



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10. Planck's constant h , speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M . Then the correct option(s) is (are)

- (A) $M \propto \sqrt{c}$ (B) $M \propto \sqrt{G}$ (C) $L \propto \sqrt{h}$ (D) $L \propto \sqrt{G}$

Answer:

Here we need to represent length (L) and Mass(M) in terms of h , c and G and need to find out how L and M dimensionally related to h, c & G .

We know that $\lambda = \frac{h}{mv}$, $h = \lambda mv$ or $[h] = [LMLT^{-1}] = [ML^2T^{-1}]$

$[c] = [LT^{-1}]$

$F = G \frac{m_1 m_2}{r^2}$ or $G = \frac{Fr^2}{m_1 m_2} = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

Writing L dimensionally in terms of h , c & G

$[L] = [h^x c^y G^z]$

$[M^0 L^1 T^0] = [ML^2 T^{-1}]^x [LT^{-1}]^y [M^{-1} L^3 T^{-2}]^z$

$[M^0 L^1 T^0] = [M^{x-z} L^{2x+y+3z} T^{-x-y-2z}]$

Now equating powers of M , L and T

$x - z = 0, x = z \rightarrow (1)$

$2x + y + 3z = 1$

putting (1) or $5x + y = 1 \rightarrow (2)$

or $-x - y - 2z = 0$ putting (1)

$-3x - y = 0$ or $3x + y = 0 \rightarrow (3)$

from (2) & (3) $2x = 1$

or $x = \frac{1}{2}$, so $z = \frac{1}{2}$

and from (2) $y = 1 - 5x = 1 - \frac{5}{2} = -\frac{3}{2}$

Therefore $[L] = [h^{1/2} c^{-3/2} G^{1/2}]$

$L \propto \sqrt{h}$, $L \propto \frac{1}{c\sqrt{c}}$, $L \propto \sqrt{G}$, option (C), (D) is correct

Writing M dimensionally in terms of h , c & G

$[M] = [h^x c^y G^z]$

$[M^1 L^0 T^0] = [ML^2 T^{-1}]^x [LT^{-1}]^y [M^{-1} L^3 T^{-2}]^z$

$[M^1 L^0 T^0] = [M^{x-z} L^{2x+y+3z} T^{-x-y-2z}]$

Now equating powers of M , L and T

$x - z = 1, z = x - 1 \rightarrow (3)$

$2x + y + 3z = 0$ putting (3)

$5x + y = 3 \rightarrow (4)$ $-x - y - 2z = 0$

or $-x - y - 2(x - 1) = 0$

or $-3x - y = -2 \rightarrow (5)$

from (4) and (5) $2x = 1$ or $x = \frac{1}{2}$, $z = -\frac{1}{2}$

and $y = -\frac{3}{2} + 2 = \frac{1}{2}$

Therefore $[M] = [h^{\frac{1}{2}} c^{\frac{1}{2}} G^{-\frac{1}{2}}]$,

$M \propto \sqrt{h}$, $M \propto \sqrt{c}$, $M \propto \frac{1}{\sqrt{G}}$, Option (A) is also correct.

Correct answer is (A), (C) & (D)